THE GRADUATE INSTITUTE GENEVA

CENTRE FOR INTERNATIONAL ENVIRONMENTAL STUDIES

Research Paper 67 **| 2021**

CAN ADJUSTMENT COSTS IN RESEARCH DERAIL THE TRANSITION TO GREEN GROWTH?

Laura NOWZOHOUR

Can adjustment costs in research derail the transition to green growth?*

Laura Nowzohour†

May 2021

Abstract

Adjustment costs are a central bottleneck of the real-world economic transition essential for achieving the sizeable reduction of greenhouse gas (GHG) emissions set out by policy makers. Could these costs derail the transition process to green growth, and if so, how should policy makers take this into account? I study this issue using the model of directed technical change in Acemoglu, Aghion, Bursztyn, and Hemous (2012), AABH, augmented by a friction on the choice of scientists developing better technologies. My results show that such frictions, even minor, materially affect the outcome. In particular, the risk of reaching an environmental disaster is higher than in the baseline AABH model. Fortunately, policy can address the problem. Specifically, a higher carbon tax ensures a disaster-free transition. In this case, the re-allocation of research activity to the clean sector happens over a longer but more realistic time horizon, namely around 15 instead of 5 years. An important policy implication is that optimal policies do not act over a substantially longer time horizon but must be more aggressive today in order to be effective. In turn, this implies that what may appear as a policy failure in the short-run — a slow transition albeit aggressive policy — actually reflects the efficient policy response to existing frictions in the economy. Furthermore, the risk of getting environmental policy wrong is highly asymmetric and 'robust policy' implies erring on the side of stringency.

Keywords: green growth, endogenous growth, directed technical change, induced innovation, environmental policy, adjustment costs.
JEL-Classification: O33, O44, Q30, Q54, Q56, Q58

*I am very grateful to Cédric Tille for his invaluable guidance and support throughout this project. I thank David Hémous for his valuable comments and suggestions along the way. Moreover, I thank Joëlle Noailly, Julien Daubanes, Eugenie Dugoua, Todd Gerarden, Weiliang Tang as well as participants of the SSES Annual Congress 2018, the Workshop on Climate Economics, Innovation and Policy 2019, and the Gerzensee Alumni Conference 2020 for helpful suggestions and comments. This study is supported by the Swiss National Science Foundation within the framework of the National Research Programme 'Sustainable Economy: resource-friendly, future-oriented, innovative' (NRP 73), Grant-No 407340-172395.

†Centre for International Environmental Studies and Department of International Economics, Graduate Institute Geneva. Email: laura.nowzohour@graduateinstitute.ch.

1 Introduction

Limiting climate change is one of the most pressing economic policy challenges of our times. It is commonly understood that achieving the sizeable reductions in greenhouse gas emissions set out by policy makers hinges on a rapid switch of economic activity towards cleaner ways of production. From an economic point of view, this requires a massive capital reallocation towards the development and adoption of cleaner technologies as well as an adequate pricing of polluting activities. The directed technical change literature points to the central role played by R&D subsidies to reach a timely and large-scale switch of research activity as well as carbon taxes to simultaneously limit pollution (Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Lemoine and Rudik, 2017).

However, re-orienting research across sectors may be subject to significant bottlenecks. This paper assesses whether adjustment frictions have a material impact on the transition towards clean technologies. In particular, whether they merely delay it or slow it so much as to push the economy into an environmental disaster. In the real world, green innovation can be driven by existing innovators who switch from polluting sectors into clean ones or by new market entrants. When considering the former, a highly educated and trained subsample of scientists, a frictionless transfer across technologies seems unlikely — scientists who innovate are often highly specialized experts in their field with a knowledge stock at least partially made up of technology-specific skills. In line with this narrative, Deming and Noray (2020) find that skills in Science, Technology, Engineering and Mathematics, the source of most innovations, depreciate fast.[1] Scientists may therefore find it unattractive to switch sectors due to prohibitive adjustment costs. When considering new market entrants, adjustment costs can be rationalized through higher entry barriers for new green innovators at the micro level (Kruse, Mohnen, Pope, and Sato, 2020) or inital financing constraints (Noailly and Smeets, 2019). In other words, the type of human capital necessary to kick-start a large-scale transition to clean technologies is not easily built over night.

The existing theoretical directed technical change literature has largely circumvented the question of whether and how scientists face adjustment costs in their transition across sectors. The standard models assume that scientists follow profit signals and that a switch of all research activity from one sector to another may take place within a relatively short time period.[2] However, the existing empirical evidence discussed above points to several

[1] A parallel can be drawn to the context of production workers where the existing evidence points to a skill mismatch between disappearing jobs in polluting industries and emerging ones in cleaner industries, not least because the latter require on average a higher level of cognitive skill, more formal education, more work experience and more on-the-job training than non-green jobs (Marin and Vona, 2019; Consoli, Marin, Marzucchi, and Vona, 2016; Vona, Marin, Consoli, and Popp, 2018).

[2] Some papers have included decreasing returns to research (Acemoglu, Aghion, Barrage, and Hémous, 2019; Greaker, Heggedal, and Rosendahl, 2018). Note that this is different from an adjustment friction which pertains to the *flow* of researchers across sectors, not to the level.

conceivable micro mechanisms that seem to suggest that green innovators in reality are faced with substantial adjustment costs at the macro level. Marginal profit incentives in the green sector are therefore unlikely to generate the type of rapid mass movement of innovation activity implied by these models. In fact, including adjustment costs materially changes their policy implications and motivates a discussion of the robustness of 'optimal' environmental policy in disaster prevention. Moreover, the extension sheds light on the critical inter-generational trade-off involved in the transition to green growth so prevalent in public policy discussions.

In the workhorse directed technical change model of Acemoglu, Aghion, Bursztyn, and Hemous (2012), henceforth AABH, the final good is produced based on two competing technologies, one polluting and one clean. Households consume a composite final good and do not internalize the environmental degradation this may be causing (environmental externality). Scientists 'build on the shoulders of giants' and innovate upon the technology which offers them a higher expected profit (path-dependence). In laissez-faire, an initial technological advantage of dirty over clean technologies leads to innovation exclusively taking place in the dirty sector so that the economy ends up in an environmental disaster. Fortunately, a benevolent social planner can implement an optimal policy package, consisting of a temporary clean research subsidy as well as a temporary carbon tax, such that all scientists switch from the dirty to the clean sector within five years and a disaster can be averted.

In this paper, I propose a simple way to augment AABH with a convex adjustment cost in research. The friction under study is one that taxes the innovation probability of scientists who newly enter the green sector, a cost which is quadratic in how many other scientists reciprocate this move simultaneously.[3] In doing so, I assume that a scientist who is new in a sector today is less likely to obtain a current-period patent as compared to a scientist in the same sector who has already been there in the previous period. Furthermore, this hampering effect is stronger, the more scientists choose a new sector at the same time — the anticipation of which discourages them from switching in the first place. Note that this friction encompasses different conceivable micro mechanisms at play, though they are not specifically modeled.[4]

The first result is that the socially optimal equilibrium in AABH is not robust to the

[3]The adjustment cost may be thought of a classical 'stepping on the toes' friction à la Jones and Williams (2000) but one that pertains to the *flow* of researchers across sectors rather than their level within a given sector.

[4]This friction is consistent with a story where existing scientists switch sectors (but are initially less likely to innovate due to the slow build-up of the specific knowledge required to do so) as well as with one where new entrants choose to innovate in a sector (but e.g. face financing constraints and are therefore initially less likely to innovate). There is no distinction between switching scientists and new scientists as the total amount of research is fixed and scientists merely choose which sector to invest in, not whether.

inclusion of an adjustment friction because the optimal policy implements a knife-edge case where even minor adustments to the model can lead to materially different results. To see this, consider the case of a benevolent social planner who is oblivious to the presence of adjustment costs in reality, i.e. her view of the world is AABH. Faced with an environmental externality, the planner implements a seemingly optimal policy toolkit, that is, a policy toolkit derived from a model without adjustment costs where scientists are *marginally* inclined to do research in the clean sector. However, on this knife-edge, the same toolkit will be insufficient to bring about the switch of innovation activity from the dirty to the clean sector necessary to avoid a disaster in the AABH model *with* the adjustment cost because scientists are no longer willing to switch. The subsequent self-perpetuating model dynamics typical for directed technical change models are unforgiving and proliferate this one-time miss into the future. Therefore, on the knife-edge, getting policies wrong by just a tiny bit on the side of laxity completely breaks the socially optimal equilibrium. In other words, if the world is not exactly as modeled in AABH, the optimal policy derived therein leads to an environmental disaster. This implies that too lax environmental policy intervention is completely ineffective — a cautioning result for policy makers that the most efficient policy may not be the most desireable one in the sense of policy robustness, i.e. allowing for a reasonable error margin, especially when the threat of cascading tipping points in our natural system looms (Jensen and Traeger, 2016; Lemoine and Traeger, 2014).

The second result is that more aggressive and front-loaded policies can fix this and restore the social optimum. First, the decreasing returns to *new* research induced by the adjustment friction lead to markedly slower equilibrium dynamics as compared to AABH. Under the friction, it is socially optimal to *gradually* switch all research activity to the clean sector — within about 15 years, depending on the strength of the friction, rather than within five years as in AABH. The more striking finding is that the social planner enacts a substantially *higher and front-loaded* research subsidy and carbon tax to implement a *slower* transition. This may seem counter-intuitive (and difficult to sell politically) but is actually in line with the logic of policy makers having to lean into the uphill battle of inducing producers to utilize *fewer* polluting inputs than they would like to *given* that innovation continues for longer in the dirty sector due to the adjustment friction. This result emphasizes the pertinence of the stringency *and* timing of environmental policy because delaying the transition is both, extremely costly (due to directed technical change) and dangerous (due to a limited carbon budget).[5] An important policy implication here is that the risk of getting environmental policy wrong is highly asymmetric. When policy makers are too lax, the carbon tax is extremely inefficient as it fails to prevent an environmental disaster but is economically distortionary nonetheless — a lose-lose. 'Robust policy' thus

[5]A point that has already been analyzed extensively in AABH is that a delayed transition to the clean sector is extremely inefficient under directed technical change. However, another important point is that the carbon budget is akin to a time constraint on tightening environmental policy because when policy remains too lax, the economic dynamics exhaust the carbon budget and everybody dies.

needs to take account of the reasonable upper bound of frictions and, in case of doubt, implies erring on the side of stringency.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 outlines my augmented version of the AABH model. Section 4 discusses the social planner equilibrium. Section 5 derives the optimal policy and discusses how the social optimum here is different from the one in AABH. Section 6 provides some results of a numerical simulation under various scenarios and discusses their policy implications. Section 7 concludes. Appendix A contains a sensitivity analysis of the long-run quality of the environment. Appendix B and C contain the remaining derivations.

2 Literature

Modern economic growth theory posits that the main driver of long-run growth is technological progress resulting from intentional investment choices of scientists (Grossman and Helpman, 1994). The importance of technology-specific progress has been underlined by Atkinson and Stiglitz (1969) who criticize the use of the aggregate production function shifter in macroeconomic models of technological change. Technology-specific progress in models of learning-by-doing where the benefits of learning are external to the firm motivate the use of subsidies or other investment-enhancing policies (e.g Arrow, 1962). According to Atkinson and Stiglitz (1969), these policies should subsidize the "right" technology instead of broad industries which may be ineffective. Which technology to innovate upon then becomes a dynamic choice of the firm incorporating current and future expected factor prices as learning-by-doing or research activities both lead to an increased stock of knowledge that is specific to a certain technology. This argument also implies that technological progress is path-dependent.

Synthesizing elements of technology-specific progress and the departure from perfect competition in innovation, Acemoglu (2002) goes one step further in discussing the determinants of *factor-biased* technical change, i.e. technical change that alters the marginal product of one factor more than that of other factors.[6] In such a setting, the direction of technical change is motivated by profit objectives of researchers and thus depends on the direction of this bias. There are two competing mechanisms at play. The *price effect* implies that technical change will be directed towards scarcer and thus more expensive factors as the resulting innovation will carry a higher price while the *market size effect* implies that technical change will be directed towards more abundant factors because (due to increasing returns to scale in the research sector) once a machine is invented, its profits are rising in the number of workers using it, i.e. in its potential market. The relative

[6]The paper synthesizes Acemoglu (1998), Acemoglu (2003a), Acemoglu (2003b) and Acemoglu and Zilibotti (2001), and Kiley (1999).

strength of these effects is determined by the elasticity of substitution between different factors.

A more recent strand of literature has applied this theory to climate change (Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Hémous, 2016; Greaker, Heggedal, and Rosendahl, 2018; Aghion, Hepburn, Teytelboym, and Zenghelis, 2014; Acemoglu, Akcigit, Hanley, and Kerr, 2016; Lemoine, 2017; Fried, 2018; Acemoglu, Aghion, Barrage, and Hémous, 2019). The directability of technical change in such models has novel implications for optimal policy. In the seminal paper by Acemoglu, Aghion, Bursztyn, and Hemous (2012), for a certain range of parameters, government intervention no longer has to be perpetual and deter economic growth forever in order to prevent natural disasters (as it would have to be in growth models without directed technical change).[7] Rather, for a high elasticity of substitution, the policy must be in place just long enough to allow the economy to move towards a state in which investments into the clean sector are more productive than those in the dirty sector in absence of government intervention. This transition happens via the re-allocation of research efforts by scientists towards the clean sector leading to a productivity catch-up of the latter. Therefore, intervention is more costly the larger is the productivity gap between the two sectors and thus optimal policy calls for a strong and immediate government intervention.

The elegance of this framework has attracted a lot of positive attention and criticism, some of which has been addressed by the ensuing literature. Hourcade, Pottier, and Espagne (2012) question the validity of the parameter choices in AABH and object to the lack of tipping points in the climate part of the model. They show that when parameters of their choice are used in the simulations, the policy implications change drastically.[8] Mattauch, Creutzig, and Edenhofer (2015) show that optimal policy has to be permanent in a version of AABH where directed technical change is substituted with a learning-by-doing mechanism. Greaker and Heggedal (2012) point out that a patent horizon of one period *only* diminishes the ability of carbon taxes to fully tackle the environmental externality because future carbon taxes do not effectively induce clean innovation today. The authors show that when the patent horizon is extended to infinity, a research subsidy ceases to be as important to implement the socially optimal equilibrium because scientists are no longer myopic.[9] Greaker, Heggedal, and Rosendahl (2018) on the other hand, emphasize the crucial role of R&D subsidies in fostering clean innovation by reinstating an intertem-

[7]It should be noted that although policy intervention is not perpetual, the time horizon for carbon taxes in AABH under the parameters used in this paper is still 200 years.

[8]The fact that policy implications change drastically under different parameter values is discussed at length in AABH but the authors simulate a version of the model at their preferred parameter values which does not correspond to the preferred values of their critics.

[9]More specifically, this is because scientists are able to reap the future benefits of redirecting their research efforts to the clean sector within the patent lifetime (Gerlagh, Kverndokk, and Rosendahl, 2014).

poral clean research externality.[10] Overall, this literature points to the sensitivity of the results drawn from AABH to model specifics which echoes a central finding of this paper. Moreover, this is the case because the results are a knife-edge case.

This paper contributes to the growing literature on endogenous and directed technical change models applied to the environment in two ways. First, this paper assesses the robustness of the social optimum derived in AABH to the inclusion of an unobserved friction, i.e. model misspecification risk. To my knowledge, this is the first paper looking at this specific issue in the context of directed technical change models. Second, the paper computes the social optimum under an adjustment friction, analyzes how this differs from the AABH baseline social optimum and what it implies for policy makers. While some existing papers include decreasing returns to research, i.e. a cost increasing in the *level* of scientists at the sector level (e.g. Hémous, 2016; Acemoglu, Aghion, Barrage, and Hémous, 2019; Greaker, Heggedal, and Rosendahl, 2018), this paper, to my knowledge, is the first to consider the role played by adjustment frictions, i.e. a cost increasing in the *flow* of scientists across sectors.

As green innovation can be driven by existing innovators who switch from polluting sectors into clean ones or by new market entrants, I can motivate the aggregate adjustment cost through various micro mechanisms. These are neither specifically modeled, nor does this paper speak to which of them is important in explaining aggregate outcomes as the model is consistent with any of them. Still, it may be useful to fix ideas and motivate what could be behind this aggregate friction.

One such micro mechanism is a potential skill mismatch between green jobs and those it replaces. While the green skills literature focuses mainly on the potential skill mismatch *across* different labour market segments, i.e. manual labour versus managerial tasks, this paper focuses on adjustment frictions *within* the market segment of highly specialized scientists, i.e. innovation.[11] While there may be some easily transferable technical skills, it

[10]More specifically, the authors augment the previous framework by decreasing returns to the *level* of research as in Jones and Williams (2000) as well as the possibility that a patent gets replaced by a better version. Both frictions raise the intertemporal research externality and strengthen the case for clean R&D subsidies. Particularly, decreasing returns to research prevent extreme outcomes where either *all* research is done in the dirty sector or *none at all* — a characteristic that AABH is notorious for. The risk of replacement on the other hand leads to an 'accelerating replacement effect' because, the more scientists enter the clean sector, the more likely it is that an innovation will be superseded by a better one. Both factors raise the wedge between the private and social benefit of clean research.

[11]Consoli, Marin, Marzucchi, and Vona (2016) find that green jobs in the US require on average a higher level of cognitive skill, more formal education, more work experience and more on-the-job training than non-green jobs. Vona, Marin, Consoli, and Popp (2018) find that the American Recovery and Reinvestment Act has lead to long-term job gains only, except for places where the skillset of green jobs overlapped with the ones of pre-existing local occupations (here: manual labour). Marin and Vona (2019) find that carbon prices may reduce jobs particularly in the area of manual labour. This suggests that one reason why green

remains questionable that a scientist who is likely to innovate upon a specific technology today has sufficiently generalizable skills to be *immediately equally likely* to undertake an innovation on a different technology. For example, Deming and Noray (2020) show that in the area of science, technology, engineering and mathematics, the earnings premium of graduates drops fast and that gaining experience causes the same individuals to leave occupations that are fast-changing. Therefore, it may well be that the general story of adjustment costs hampering movements from polluting to clean sectors is also valid for the highly specialized segment of innovating scientists.

Another such micro mechanism are entry barriers for new inventors. Although market entry and exit in research overall is not specifically modelled here (the total amount of research is fixed), there may be market entry and exit in each period at the sector level. The existing evidence in the innovation literature points to new entrants driving most of the switch of innovation activity from dirty to clean technologies. Noailly and Smeets (2015) find that it is mainly through market entry of specialized renewable-energy (RE) firms, following an increase in RE market size, that the technology gap between RE and fossil-fuel technology closes. There is also some evidence for within-firm R&D shifting from fossil fuels to renewable-energy research in response to market incentives (Popp and Newell, 2012). Assuming that the bulk of new green innovation naturally comes from new entrants does not take away from the bottleneck posed by entry barriers. Kruse, Mohnen, Pope, and Sato (2020) show that firms producing green goods and services have higher capital asset requirements per unit of sales for up to two years after starting to generate green revenues. They are also more likely to face financing constraints as they rely more heavily on internal finance (Noailly and Smeets, 2019). Therefore, even if the transition is driven by new entrants, adjustment costs in the form of entry barriers in the green sector seem to be a relevant feature of the transition process.

To summarize, this paper augments the existing literature on endogenous and directed technical change applied to the environment by providing insights on the effects of an aggregate adjustment friction on the equilibrium outcome. The friction is consistent with various mechanisms documented in the empirical literature on green skills and innovation and encompasses the case of green innovation stemming from scientists who switch from polluting to clean sectors and the case of new entrants. A first contribution is to assess the robustness of the social optimum in AABH to unobserved frictions, i.e. model misspecification risk. A second contribution is to analyze of how the socially optimal dynamics change in the presence of the adjustment costs and what this implies for policy makers.

---

stimulus packages have had limited effects on short-term job gains to-date may be due to unaccounted adjustment frictions in the labour market (Noailly, Popp, and Vona, 2020).

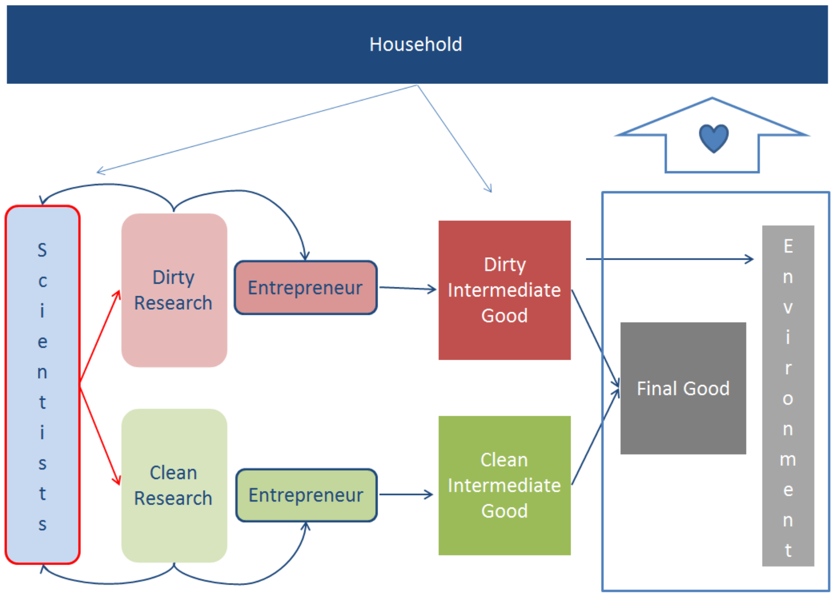

Figure 1: Illustration of Acemoglu et al. (2012)

3 Framework

The framework under study takes the workhorse model in Acemoglu, Aghion, Bursztyn, and Hemous (2012), AABH, as a baseline. My contribution is to augment it with an aggregate adjustment friction in research which is indicated in **bold** for ease of interpretability of its effects relative to the AABH baseline. The model considers a discrete-time economy with an infinite horizon. Figure 3 illustrates the model graphically. All derivations can be found in Appendix B.

3.1 Households

The representative household draws utility from consumption of a unique final good at time t, C_t, and the quality of the environment at time t, S_t, which can collapse. Its preferences are described by a CRRA utility function:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{(\phi(S_t)C_t)^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $\rho > 0$ is the discount rate, $\sigma \geq 0, \sigma \neq 1$ is the coefficient or relative risk aversion and $\phi(S_t)$ is the cost of environmental degradation, defined and parametrized in Section 6. The budget constraint is:

$$C_t = \underbrace{w_t(L_{c,t} + L_{d,t})}_{\text{Labour income}} + \underbrace{\Pi_{c,t} + \Pi_{d,t}}_{\text{Research profits}} + \underbrace{\tau_t Y_{d,t}}_{\text{Carbon tax rebate}}$$

Households comprise of workers, scientists and entrepreneurs and draw income from labour supply of the workers and the profits of entrepreneurs.

3.2 The quality of the environment

The quality of the environment evolves on an interval between 0 and $\overline{S}$, where the latter corresponds to the best possible state of nature absent any human intervention and the former corresponds to an environmental disaster, i.e. a point of no return. These dynamics are captured by the following difference equation:

$$S_{t+1} = \begin{cases} 0 & \text{if } S_t = 0 \\ -\xi Y_{d,t} + (1+\delta)S_t & \text{if } 0 < S_t < \overline{S} \\ \overline{S} & \text{if } S_t = \overline{S} \end{cases} \tag{2}$$

where ξ corresponds to the strength of the environmental externality, that is, the extent to which production of the dirty intermediate good, $Y_{d,t}$, negatively affects the quality of the environment, and δ corresponds to the rate of environmental regeneration. As in AABH, an environmental disaster is defined as any period in which $S_t = 0$ for some $t < \infty$.

3.3 Production

The final good is produced under perfect competition and using an aggregate of clean and dirty intermediate goods according to a standard CES production function using clean technology, $Y_{c,t}$, or dirty technology, $Y_{d,t}$:

$$Y_t = \left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where $\varepsilon \in (0, \infty)$ is the elasticity of substitution between clean and dirty inputs. Throughout this paper, analogously to AABH, I assume that $\varepsilon > 1$, which means that clean and dirty inputs are gross substitutes in the production of the final good. Production of the dirty good adversely affects the quality of the environment (more on this below).

Clean and dirty intermediates are produced symmetrically using sector-specific labour, $L_{j,t}$, or sector-specific machines of sector- and machine-specific quality, $A_{j,i,t}$, where $j = \{c, d\}$:

$$Y_{j,t} = L_{j,t}^{1-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha} di \tag{4}$$

where $\alpha \in (0,1)$ is the intermediate-good elasticity of the machine input. The total mass of workers is normalized to one, with a fraction $L_{j,t}$ working in sector $j = \{c,d\}$ so that labour market clearing implies:

$$L_{c,t} + L_{d,t} \leq 1, \tag{5}$$

Market clearing of the final good implies:

$$C_t + \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) = Y_t \tag{6}$$

where $\psi = \alpha^2$. This means that what is produced by firms can either be consumed by the household or be used for production of new machines in either of the two sectors at a real cost of ψ units of the consumption good. Note that there is no investment in this model.

3.4 The innovation possibilities frontier

The growth engine of the model is the profit-maximizing behaviour of scientists. The innovation possibilities frontier (IPF) is as follows. Each scientist decides every period whether to direct her research effort towards the clean or dirty sector and is then allocated to at most one machine for improvements. She succeeds in innovating with probability $(1 - \boldsymbol{\Psi^i_{j,t}})\eta_j$, where $j = \{c,d\}$. In case of success, the quality of the machine improves by a factor of γ and she obtains a current-period patent allowing her to earn monopoly profits by renting out the improved machine to intermediate good producers of the respective sector. If innovation fails, the machine of the old quality is randomly allocated to an entrepreneur who operates it for the current period at previous-period's profits. All of this takes place within one period and is detailed below.

As with labour, the total supply of research is fixed at one, but here the sectoral allocation is endogenous and responds to a combination of factors discussed below. A fraction $s_{j,t}$ works in sector $j = \{c,d\}$ so that market clearing implies:

$$s_{c,t} + s_{d,t} \leq 1 \tag{7}$$

The average sectoral productivity is defined as an aggregate over all individual machine productivities:

$$A_{j,t} \equiv \int_0^1 A_{j,i,t} di \tag{8}$$

3.4.1 Technological progress

At the individual machine level, quality evolves as follows: conditional on being matched with a scientist ($s^i_{j,t} = 1$), the probability of an improvement is $\eta_j(1 - \Psi^i_{j,t})$ and in case of success, the machine improves by a factor of γ, i.e. from $A_{j,i,t-1}$ to $(1+\gamma)A_{j,i,t-1}$, where $\Psi^i_{j,t}$ is a real adjustment cost detailed below. In case no scientist gets matched with the machine, the quality remains at its previous-period level and the machine lays idle for this period. This is captured by the following difference equations:

$$
\begin{aligned}
A_{j,i,t}|_{s^i_{j,t}=1} =& A_{j,i,t-1} + \underbrace{\eta_j(1 - \boldsymbol{\Psi^i_{j,t}})}_{\text{Prob. of success}} \gamma A_{j,i,t-1} + \underbrace{(1-\eta_j)\,(1 - \boldsymbol{\Psi^i_{j,t}})}_{\text{Prob. of failure}} \times 0 \\
=& \left(1 + \eta_j(1 - \boldsymbol{\Psi^i_{j,t}})\gamma\right) A_{j,i,t-1} \\
A_{j,i,t}|_{s^i_{j,t}=0} =& A_{j,i,t-1}
\end{aligned}
$$

Therefore, the quality improvement at machine level is:

$$
A_{j,i,t} = \left(1 + \eta_j \boldsymbol{(1 - \Psi_{j,t})} \gamma s^i_{j,t}\right) A_{j,i,t-1} \tag{9}
$$

To aggregate at the sector level, integrate over all machines by plugging (9) in (8) to obtain:

$$
A_{j,t} = \left(1 + \eta_j \boldsymbol{(1 - \Psi_{j,t})} \gamma s_{j,t}\right) A_{j,t-1} \tag{10}
$$

where

$$
\Psi_{j,t} \equiv \int_0^1 \Psi^i_{j,t} di
$$

Scientists can only target a sector but not a machine and thus orient their choice at the aggregate expected machine quality at sector level in (10). Each sector inherits its previous-period average quality which reflects the intertemporal spillovers of research. Each machine with a scientist will be improved by a factor γ with probability $\eta_j(1 - \Psi_{j,t})$.

3.4.2 Adjustment costs

Departing from AABH, I assume that there is a real cost involved for any scientist who newly enters a sector, one that is convex in the aggregate inflow of new scientists. This congestion friction is modeled as a reduction of the scientist's current-period innovation probability in the new sector. The relevant parameter space here focuses on the case where scientists switch from the dirty to the clean sector so the cost can be written as:

$$\boldsymbol{\Psi^i_{c,t}} = \begin{cases} \left(\frac{\kappa}{2}\right) \Delta^2_{s_{c,t}} & \text{if } \Delta_{s^i_{c,t}} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where $\Delta_{s_{c,t}} = s_{c,t} - s_{c,t-1}$ is the aggregate inflow of new scientists to the clean sector, $\Delta_{s^i_{c,t}} = s^i_{c,t} - s^i_{c,t-1}$ with $s^i_{c,t} = \{0, 1\}$ indicates whether the scientist newly enters the clean sector or not, and $\kappa \in [0, \infty)$ is the strength of the adjustment cost. Note that the cost only applies ($\boldsymbol{\Psi^i_{c,t}} > 0$) in periods where there is an aggregate inflow of new scientists into the clean sector ($\Delta^2_{s_{c,t}} > 0$), i.e. it is a short-run adjustment friction. Moreover, it only taxes the clean sector with new entrants without subsidizing the dirty one with exiters ($\Delta_{s^i_{c,t}} > 0$).

Upon newly entering a sector, scientists face an adjustment cost which is quadratic in the aggregate scientist reallocation and intuitively implies that it becomes disproportionately more costly for them to enter a new sector when many others do so simultaneously ($\Psi_{j,t} > 0$). At any point in time, her individual choice, $s^i_{j,t}$, is thus optimal *given* the aggregate scientist allocation in t.

4 The Equilibrium

This section describes the scientist allocation *given* a set of policies to address the market failures in the model. First, a carbon tax is implemented to tackle the environmental externality which manifests itself the unintended pollution resulting from household consumption of the composite good — when and if this is made up of the dirty intermediate to some extent. Moreover, a clean research subsidy is implemented to account for the intertemporal knowledge spillovers. This externality results from the short patent horizon of one period only which prevents entrepreneurs from reaping the future benefits of their research efforts and therefore leads to an under-supply of research with respect to what is socially desirable. Finally, as in AABH, it is assumed throughout that a subsidy to the use of all machines to tackle the monopoly distortion in intermediate goods production is in place so that any deviation of the socially optimal outcome from laissez-faire is due to the previous two externalities.

I first characterize the equilibrium for given productivity parameters and then analyze the direction of technical change, specifically how this differs from the frictionless baseline case in AABH. For ease of readability, I code the expressions as follows: black corresponds to the AABH laissez-faire case. Gray augments the black expressions to correspond to the AABH planner solution, i.e. including the carbon tax and the clean research subsidy. The adjustment cost in clean research is inserted in **bold** so that all expressions nest the AABH baseline case for $\boldsymbol{\Psi^{i}_{c,t}}(\kappa) = 0$ whenever $\kappa = 0$. All further derivations are in Appendix B.

Definition 1 *An equilibrium is given by a sequence of wages, $\{w_t\}$, input prices, $\{p_{j,i,t}\}$, machine demands, $\{x_{j,i,t}\}$, input demands, $\{Y_{j,t}\}$, labour demands by input producers, $\{L_{j,t}\}$, research allocations, $\{s_{j,t}\}$ and environmental qualities, $\{S_t\}$, such that in each period t:*

1. *$(p_{j,i,t}, x_{j,i,t})$ maximizes profits of the producer of machine i in sector j,*
2. *$L_{j,t}$ maximizes profits by producers of input j,*
3. *$Y_{j,t}$ maximizes the profits of final good producers,*
4. *$s_{j,t}$ maximizes the expected profit of a researcher at date t,*
5. *w_t and $p_{j,t}$ are such that labour and input markets clear, respectively, and*
6. *S_t evolves according to (2).*

I focus on the parameter space where scientists, if at all, switch from the dirty to the clean sector in response to policy intervention. To do so, I assume that the initial relative levels of productivity in both sectors be such that in laissez-faire, innovation takes place exclusively in the dirty sector. This is an augmented version of Assumption 1 in AABH and is formally derived in Appendix B.4.6.

4.1 The equilibrium given technology

Starting with the profit maximization of the final good producer, I show in Appendix B.1 that the relationship between relative demand of the two intermediates and their relative prices depends on the elasticity of substitution:

$$\frac{Y_{c,t}}{Y_{d,t}} = \left(\frac{p_{c,t}}{(1+\tau_t)p_{d,t}}\right)^{-\varepsilon} \tag{12}$$

When $\varepsilon > 0$, relative demand drops in response to an increase in relative prices. When the two intermediates are gross substitutes ($\varepsilon > 1$), the final good producer is more sensitive to price movements in adjusting her relative demand towards the good which is cheaper. On the contrary, when the two intermediates are gross complements ($\varepsilon < 1$), relative demand drops if $\varepsilon > 0$ but is less sensitive to an increase in relative prices and rises if $\varepsilon < 0$ because the two intermediates are preferably used in fixed proportions.

To study the evolution of average productivities in the two sectors, consider the prospective profits of an entrepreneur in sector j (derived in Appendix B.3):

$$\pi_{j,i,t} = (1 + \mathbb{I}_{\{j=c\}}\nu_{c,t})\alpha(1-\alpha)(1-\chi)^{-\frac{\alpha}{1-\alpha}} p_{j,t}^{\frac{1}{1-\alpha}} L_{j,t} A_{j,i,t} \tag{13}$$

where $\nu_{c,t}$ is the optimal clean research subsidy at time t, $\mathbb{I}_c$ is an indicator variable equal to one if $j = c$ and zero otherwise and χ is the optimal subsidy to the use of all machines.

As the scientist is allocated to a machine within sectors at random, they are only interested in the expected profits at sector level. Remember that they are successful at improving a machine by a factor of $(1 + \gamma)$ with respect to its previous-period level of productivity with a probability of $(1 - \Psi_{c,t})\eta_c$ if they switch to the clean sector and η_d if they remain in the dirty sector. Thus, the expected gain from switching to the clean sector and remaining in the dirty sector, respectively, is:

$$\begin{aligned} \Pi_{c,t} &= (1 + \nu_{c,t})(1 - (\mathbf{1} - \boldsymbol{\Psi}_{c,t}))\Pi_{c,t}^{\text{AABH laissez-faire}} \\ \Pi_{d,t} &= \Pi_{d,t}^{\text{AABH laissez-faire}} \end{aligned} \tag{14}$$

where $\Pi_{j,t}^{\text{AABH laissez-faire}} = \eta_j \alpha(1-\alpha)(1-\chi)^{-\frac{\alpha}{1-\alpha}} p_{j,t}^{\frac{1}{1-\alpha}} L_{j,t}(1+\gamma)A_{j,t-1}$ for $j = \{c, d\}$ and $\Psi_{j,t}(\kappa)$ is the adjustment cost with intensity κ. Dividing (14) for both sectors, I obtain:

$$\frac{\Pi_{c,t}}{\Pi_{d,t}} = (1 + \nu_{c,t})\,(\mathbf{1} - \boldsymbol{\Psi}_{c,t})\left(\frac{\eta_c}{\eta_d}\right)\underbrace{\left(\frac{p_{c,t}}{p_{d,t}}\right)^{\frac{1}{1-\alpha}}}_{\text{Price effect}}\underbrace{\left(\frac{L_{c,t}}{L_{d,t}}\right)}_{\text{Market size effect}}\underbrace{\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)}_{\text{Direct productivity effect}} \tag{15}$$

Note that by switching off the congestion friction ($\kappa = 0$) and setting $\nu_{c,t} = 0$, I obtain (16) and (17) in AABH from (14) and (15), respectively.

Let me briefly discuss the intuition behind the scientist choice based on (15). As in AABH, there are three forces at play: (i) the price effect, (ii) the market size effect, and (iii) the direct productivity (or 'building on the shoulders of giants') effect. First, the **price effect** reflects that scientists prefer to innovate where they expect *higher prices* because for a given unit of innovation, they earn higher profits. For $\alpha \in (0, 1)$, the input produced with the more productive machines will be relatively cheaper so this effect favors the more *backward* sector. Second, the **market size effect** reflects that scientists prefer to innovate where they expect a *larger market* (or a larger labor force) because any given unit of innovation will be met by a larger demand. For $\alpha \in (0, 1)$ and $\varepsilon > 1$, the more advanced sector attracts a relatively larger labor force and thus offers a larger market for any given innovation so this effect favors the more *advanced* sector. Lastly, the **direct productivity effect** reflects that I prefer to innovate where I benefit from a *higher pre-existing level of productivity.* This favors by definition the more advanced sector. I show these results in Appendix B.4.1.

4.2 The direction of technical change

In order to pin down the choice of the scientists and characterize the direction of technical change, we need relative expected profits as a function of predetermined variables or parameters and the scientist allocation across sectors only. I show in Appendix B.4.5 that (15) can be written as:

$$\frac{\Pi_{c,t}}{\Pi_{d,t}} = (1+\nu_{c,t})(1+\tau_t)^{\varepsilon}\, \mathbf{(1-\Psi_{c,t})} \left(\frac{\eta_c}{\eta_d}\right) \left(\frac{1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}{1+\gamma\eta_d(1-s_{c,t})}\right)^{-(1+\phi)} \left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)^{-\phi} \tag{16}$$

where $\phi \equiv (1-\alpha)(1-\varepsilon)$ without loss of generality. By disabling the friction ($\kappa = 0$) and setting policy variables to zero ($\nu_{c,t} = \tau_t = 0$), I get back to equation (18) in AABH. I can now find the optimal allocation of scientists, $s_{c,t}$, each period by using an adjusted version of Lemma 1 from AABH. While the logic is the same here, the situation is slightly more complex because, depending on κ and the previous-period scientist allocation, $s_{c,t-1}$, the relative profit function may no longer be either strictly increasing, decreasing or constant in $s_{c,t}$ but may now also be u-shaped or inverse-u shaped, giving rise to more possible equilibria. This is described more formally in Appendix B.4.5.

The allocation of scientists across sectors in each period is the central aspect of the model because the endogeneity in the direction of technical change comes from scientists choosing each period whether they attempt to improve a machine in the clean or dirty sector.[12] How does the adjustment cost, $\mathbf{(1-\Psi_{c,t})} = \left(\frac{\kappa}{2}\right)\Delta^2_{s_{c,t}}$ if scientists flow into the clean sector ($\Delta_{s_{c,t}} > 0$), alter these pre-existing mechanisms? First, note that the adjustment cost is increasing in $\Delta_{s_{c,t}}$, i.e. the more scientists switch to the clean sector today, the lower will be the probability of concurrent innovation there. Ceteris paribus, the larger is κ, the less attractive it is to switch to the clean sector. Thus, for a given level of relative profits, scientists are less willing to switch to the clean sector when $\kappa > 0$ as compared to AABH. Moreover, for a given scientist allocation, the larger is κ, the *more aggressive* policies have to be to induce a switch of research activity to the clean sector. In other words, the friction makes the job of the social planner harder and implies that: (i) policy intervention aimed at achieving the AABH outcome needs to be *more aggressive* compared to the baseline AABH policy, and, (ii) because switching is costly, it may be socially optimal to implement a slower switch of research activity towards the clean sector (more on this later). I describe the structure of optimal policy under the friction in the next section.

[12] Note that the endogeneity in the growth process here pertains to the allocation across sectors, that is, the *direction* of technical change and not the long-run *growth* of technical change as the total supply of researchers is fixed. In the long-run, the economy grows at a rate $\gamma\eta_j$ depending on which technology j is dominant in innovation.

5 Optimal Policies

In this section, I present the socially optimal allocation involving: (i) a lump-sum carbon tax on the use of the dirty machine in order to discourage use of the polluting intermediate good, (ii) a subsidy to clean research to encourage clean research activity and account for the intertemporal research externality, and (iii) a subsidy to the use of all machines to correct for the monopoly distortion in the machine sector — with a particular focus on what the presence of the adjustment friction changes with respect to the socially optimal policy and allocation in AABH. I assume that the subsidy to the use of all machines derived in Appendix C.2 is present throughout. All further derivations can be found in Appendix C. I first define what constitutes a socially optimal equilibrium.

Definition 2 *The socially optimal allocation is a dynamic path of consumption, C_t, environmental quality, S_t, final good production, Y_t, input productions, $Y_{j,t}$, machine production, $x_{j,i,t}$, labour allocations, $L_{j,t}$, scientist allocations, $s_{j,t}$, and machine qualities, $A_{j,i,t}$, that maximizes the intertemporal utility of the representative household, (1), subject to the aggregate production function, (3), the law of motion of environmental quality, (2), intermediate good production, (4), the resource constraint, (6), labour market clearing, (5), research market clearing, (7), and the law of motion of technology, (10).*

AABH show that this allocation can be implemented using a tax on the dirty intermediate good as well as a subsidy to clean research and a subsidy to the use of all machines, with all proceeds being financed and redistributed as a lump sum. This result continues to hold here but, as we shall see, the evolution of optimal policies is different in many cases.

5.1 The carbon tax

The carbon tax has the primary purpose of discouraging final good producers from using the polluting intermediate good in final good production by introducing a wedge between the marginal product of the dirty input in the production of the final good and its shadow value. I show in Appendix C.1 that this wedge, τ_t, corresponds to:

$$\tau_t^* = \left(\frac{\xi}{\hat{p}_{d,t}}\right) \frac{\frac{1}{(1+\rho)} \sum_{j=t+1}^{\infty} \left(\frac{1+\delta}{1+\rho}\right)^{j-(t+1)} \frac{\partial u(C_j,S_j)}{\partial S_j} \mathbf{I}_{\{S_j<\overline{S}\}}}{\frac{\partial u(C_t,S_t)}{\partial C_t}} \tag{17}$$

where $\hat{p}_{d,t}$ is the shadow producer price of the dirty input at time t in terms of the final good and $\mathbf{I}_{\{S_j<\overline{S}\}}$ is 1 whenever $S_{t+1}, \ldots, S_v < \overline{S}$, i.e. whenever the environment is not yet at its best possible state, and 0 otherwise.

This wedge induces the price of the dirty intermediate good to rise above its marginal cost of production, making it less attractive for final good producers to utilize it. By inducing a shift in the input portfolio of final good producers towards the clean intermediate good, the tax has a direct positive effect on the quality of the environment today. However, due to the regenerative capacity of the environment, it also has a lasting impact on the evolution of the environment into the future. These benefits of the carbon tax are captured by the numerator in (17), indicating that the tax is proportional to the strength of the environmental externality, ξ, weighted by the discounted stream of current and future marginal utility gains brought about by an extra unit of environmental improvement, accounting for the possibility that the environment may reach its best possible state absent any human intervention, $\overline{S}$, from where it cannot be further improved. The tax also has a cost in terms of consumption because it is distortionary. By driving a wedge between marginal cost and price of the dirty intermediate good, it induces final good producers to substitute a fraction of their dirty inputs with the less-developed clean intermediate. By purposefully using a less efficient input, consumption drops relative to what it could have been in the tax-free case. Therefore, the carbon tax is scaled by the current shadow producer price of the dirty input in terms of the final good and the current marginal utility of consumption, i.e. the more important the dirty intermediate input is for final good production and the more households are starved for an extra unit of consumption, the lower is the socially optimal tax. To summarize, the carbon tax optimally solves the utility-maximizing trade-off between consumption and environmental quality by weighting utility losses associated with current-period economic distortions with current and future utility gains stemming from better environmental quality.[13]

How is the optimal carbon tax here different from the one in AABH? The expression in (17) exactly corresponds to (23) in AABH. Nevertheless, there are important differences in the evolution of the optimal carbon tax in this augmented setup when and if the presence of the adjustment friction changes the socially optimal allocation of scientists across sectors from the AABH social optimum, i.e. whenever it is socially optimal to gradually switch innovation activity from the dirty to the clean sector ($\Delta_{s_{c,1}} < 1$) as opposed to a complete and immediate switch ($\Delta_{s_{c,1}} = 1$). In that case, the adjustment friction puts upward pressure on this wedge because preventing the final good producer from using the dirty intermediate in final good production, whereby it would contribute to a worsening of environmental quality, becomes costlier the larger the technology gap of the two intermediate goods is. Intuitively, it becomes costlier to induce someone to go for a less lucrative option, the more lucrative her outside option is (more on this in the numerical analysis of

[13]Note that the carbon tax also fulfills the secondary purpose of reallocating scientists from the polluting to the clean sector by lowering their prospective profits in the polluting sector. However, using the carbon tax for this purpose is inefficient as the tax would have to be excessively high, thereby over-encouraging the use of the initially inefficient clean technology, in order to fulfil this secondary mandate effectively. Instead, the task of reallocating scientists is taken up by the research subsidy. This is discussed in detail in AABH.

the model below).

5.2 The clean research subsidy

The primary purpose of the clean research subsidy is to account for the intertemporal research externality coming from the myopia of scientists who have access to a one-period patent upon successfully innovating and thus do not internalize the positive effect current innovations have on future technology growth. I show in Appendix C.3 that it is socially optimal to allocate scientists to the clean sector whenever:

$$\underbrace{\left(\frac{\eta_c}{\eta_d}\right)\boldsymbol{(1-\Psi_{c,t})}\left(\frac{1+\boldsymbol{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}{1+\gamma\eta_d s_{d,t}}\right)^{-(1+\phi)}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)^{-\phi}}_{\text{Private value of innovation in clean vs. dirty in t}}\left(\frac{\sum_{v>t}\lambda_v L_{cv}A_{cv}\hat{p}_{cv}^{\frac{1}{1-\alpha}}}{\sum_{v>t}\lambda_v L_{dv}A_{dv}\hat{p}_{dv}^{\frac{1}{1-\alpha}}}\right)>1 \tag{18}$$

where $\lambda_v = \frac{1}{(1+\rho)^v}\frac{\partial u(C_v,S_v)}{\partial C_v}$ is the household's discount factor. (18) reflects the social value of doing research in the clean sector versus the dirty sector. The first four terms correspond to the relative expected profits of doing research in the clean versus dirty sector for a scientist and are already familiar from (16). However, because scientists are myopic, they do not internalize future gains in productivity growth resulting from their innovation today as reflected by the last term. The clean research subsidy makes sure that whenever private incentives are insufficient to induce scientists to go where the social value of innovation is higher, scientists will be compensated just enough to be willing to behave in accordance with the social optimum.

Specifically, I show in Appendix C.3 that the social planner can implement the optimal subsidy, ν_t, by ensuring that whenever the optimal scientist allocation involves at least some research to be done in the clean sector ($s_{c,t} > 0$) but private incentives of scientists are inconsistent with this allocation $\left(\left.\frac{\Pi_{c,t}}{\Pi_{d,t}}\right|_{s_{c,t}=1} < 1\right)$, the optimal subsidy to clean research must be at least as high as the excess profits in the dirty sector, $\hat{v}_t$. Specifically,

$$\nu_t \geq \hat{\nu}_t \equiv \left.\frac{\Pi_{d,t}}{\Pi_{c,t}}\right|_{s_{c,t}=1} - 1, \tag{19}$$

In periods where the clean research subsidy is active, i.e. in the first periods where the reallocation of scientists must take place, scientists are on a knife-edge.[14] I show this numerically in Appendix A. The lowest possible subsidy, $\nu_t = \hat{\nu}_t$, in (19) is *just* high enough

[14] Note that the clean research subsidy is non-distortionary so the social planner could easily set $\nu_t > \hat{\nu}_t$

so that scientists do not want to leave the green sector, that is, the social planner virtually puts scientists on the edge of a cliff. If the clean research subsidy is only minimally smaller than excess profits in the dirty sector, then scientists fall off the cliff — they do not bother switching to the clean sector, innovation continues to take place exclusively in the dirty sector and the self-reinforcing dynamics lock in the dirty equilibrium which eventually leads to an environmental disaster. On this edge, however, scientists innovate in the clean sector. Until clean technologies have caught up with dirty ones, private research incentives are such that scientists would fall off the cliff in laissez-faire so the social planner has to keep subsidizing them for a number of consecutive periods until the cliff disappears. Only after clean technologies have caught up with dirty ones do scientists innovate in the clean sector in absence of the subsidy and the cliff reverses — now it is clean technologies that get locked in for the long-term.

Intuitively, one may might think of a balancing scale with a ball on it. If the ball is placed only a millimetre to the right of the centre, the scale will tilt fully in that direction, and the opposite will happen if the ball is put only slightly to the left of the centre. Assumption 1 posits that the scale is initially tilted in favour of dirty technologies. It requires the social planner's effort to simulate a situation for scientists that the scale is even. It is until the clean innovation efforts by scientists, brought about by the purposeful action of the social planner, reinforce the situation she is simulating, that her mission is completed and the balancing scale, by itself, starts tilting in favour of clean technologies.

How is the optimal clean research subsidy in here different from the one in AABH? The expression in (18) shows that for any *given* scientist allocation, the subsidy required to implement this as an equilibrium is *higher* than in the frictionless case because the friction lowers the relative expected profits in the clean sector whenever $\Delta_{s_{c,t}} > 0$ (see (14)). However, because the friction lowers the current research productivity of new clean innovators, the spillovers accruing from their research activities in periods where they switch are lower, thereby lowering the social value of clean research. Whether the equilibrium subsidy will be higher or lower than the AABH baseline is therefore ex ante unclear.

---

which would put scientists away from the cliff. In reality, implementing a subsidy is hardly considered costless so while one could make a point here that the optimal subsidy in this model only has a lower bound but can, in theory, be as a high as the modeller wishes, this is not a helpful approximation of reality. Rather, I assume that the social planner here implements the lowest clean research subsidy possible and then analyze the sensitivity of this policy to the presence of unobserved frictions.

6 Results and Policy Implications

6.1 Parameters

I now turn to quantitative results. As in AABH, one period in the model corresponds to five years and the innovation part of the model is calibrated so that the annual long-run growth rate amounts to 2%. It is further assumed that the subsidy to the use of all machines is present throughout, as this makes sure the results are not driven by monopoly distortions in the market for machines. The initial levels of technology are calibrated to a situation where this subsidy is in place, but absent optimal policy otherwise, to match the implied values of initial clean and dirty output from the production of nonfossil and fossil fuel in the world primary energy supply from 2002 to 2006.[15] All other parameter values shown in Table 1 are also used in AABH.

	Parameter	Value
Innovation		
Length of one period	dt	5 years
Probability of success in the clean sector	$\eta_c \times dt$	10%
Probability of success in the dirty sector	$\eta_c \times dt$	10%
Size of innovation	γ	1
Initial scientist allocation (not in AABH)	$s_{c,0}$	0
Production		
Elasticity of substitution b\w intermediates	ε	3
Share of machines in production	α	1/3
Cost of machines	ψ	$\alpha^2 = 1/9$
Welfare		
Coefficient of relative risk aversion	σ	2
Discount factor	$\rho \times dt$	0.005
Environment		
Disaster temperature increase since pre-ind. times	Δ_{disaster}	$6°C$
Regularization parameter of env. damages	λ	0.1443
Rate of environmental regeneration	δ	0.0118

Table 1: Parameters from AABH

[15] While the data may seem outdated, it allows for better comparability of the results in this model to the AABH baseline.

6.1.1 The cost of environmental degradation

The cost of environmental degradation, $\phi(S_t)$, is defined as in AABH. Specifically:

$$\phi(S) = \frac{(\Delta_{\text{disaster}} - \Delta(S))^{\lambda} - \lambda \Delta_{\text{disaster}}^{\lambda-1} (\Delta_{\text{disaster}} - \Delta(S))}{(1-\lambda)\, \Delta_{\text{disaster}}^{\lambda}},$$

where Δ_{disaster} is the temperature increase since pre-industrial times corresponding to a natural disaster (here: $6°C$), λ is a regularization parameter used to match this function to to Nordhaus's damage function for the temperature increase range up to $3°C$, and

$$\Delta = 3 \log_2 \left(\frac{C_{CO_2}}{280} \right) \tag{20}$$

is the temperature increase at the current CO_2 concentration in parts per million (ppm), where 280 is the atmospheric CO_2 concentration absent any human intervention. (20) implies that when the atmospheric concentration of CO_2 doubles, there will be an increase of $3°C$ in current temperatures with respect to pre-industrial temperatures. Moreover, to obtain an estimate of the disaster level of CO_2 concentration in the atmosphere, simply plug $\Delta_{\text{disaster}} = 6°C$ into (20) and solve for $C_{CO_2,\text{disaster}}$. One may now relate the model-implied evolution of the quality of the environment, S, to the atmospheric concentration of carbon by setting:

$$S = C_{CO_2,\text{disaster}} - \max\{C_{CO_2}, 280\}$$

and solving for C_{CO_2}. The rate of environmental regeneration, δ, is calibrated so that half of CO2 emissions are absorbed at current atmospheric levels.

6.1.2 The strength of the adjustment friction

The AABH baseline model ($\kappa = 0$) under the given parameters tells us that it is socially optimal to re-allocate scientists to the clean sector fully in the first period. Therefore, the strength of the adjustment friction, κ, can be calibrated by mapping it to a cost in % of initial technology. Recall from (10) that clean productivity in the first period is:

$$A_{c,1} = \left(1 + \underbrace{\eta_c (1 - \boldsymbol{\Psi_{c,1}}) \gamma s_{c,1}}_{\equiv \Omega_1} \right) A_{c,0}$$

Define Ω_t as the 'innovation efficiency' and $\hat{\Omega}_t$ as the percentage deviation of Ω_t from the AABH baseline *given* an allocation, $s_{c,1}$. Specifically:

$$\hat{\Omega}_1(\kappa) \equiv \omega_1(\kappa) - \omega_1(0) \approx -\Psi_{c,1}(\kappa)$$

where ω is the natural logarithm of Ω_t. Then, $\hat{\Omega}_t$ can be interpreted as the loss in innovation efficiency due to the presence of the friction for a given scientist allocation.

Figure 2 plots $\hat{\Omega}_1(\kappa)$ for different values of κ which span a reasonable parameter range. Intuitively, the figure can be interpreted as "the resources necessary to induce x% of innovators to switch to the clean sector within five years correspond to y% of their innovation efficiency during those first five years". Accordingly, a κ of 0.5/1/2 translates into a loss of 25/50/100% of research efficiency relative to baseline at an initial research re-allocation of *all* scientists from the dirty to the clean sector (the baseline AABH social optimum), whereas a κ of 3 translates into a loss of 100% of research efficiency relative to baseline at an initial research re-allocation of 2/3 of all scientists from the dirty to the clean sector.

Depending on the degree of specific knowledge required for clean innovation, all of these scenarios fall withing a somewhat conceivable range of scenarios.[16] Note that, almost without loss of generality, I can take $\kappa = 0.5$ (burning 25% of the clean innovation efficiency gains at a full switch within five years) as a conservative estimate for the true adjustment costs because, as Figure 3 shows, the optimal change in scientist allocation relative to the AABH baseline as a result of the friction happens early on. In other words, the model dynamics are extremely sensitive to κ for κ close to zero but the marginal impact of a higher κ on the research allocation diminishes fast.[17]

6.2 A tale of errors you really wish you hadn't made

Figure 4 describes the optimal policy derived in AABH which I call *baseline* policy ($\kappa = 0$). In Figure 6, the dotted line refers to the AABH baseline model ($\kappa = 0$) and the full line refers to the model with a very small friction ($\kappa = 0.1$), in that, it burns only 5% of the clean innovation efficiency gains at a full switch within five years, under the ABHH baseline policy. The striking result is that even such a small friction leads to a completely different equilibrium from AABH.

This is because the incentive put in place by the baseline research subsidy is insufficient to induce scientists to switch to the clean sector in the first period. As (19) shows, the

[16] Note that the socially optimal scientist allocation may, under a strong friction $\kappa > 2$, imply that clean research taxes the existing technology stock more than it adds in the first period where the switch takes place. This may be part of a socially optimal allocation if the future loss due to environmental degradation exceeds the current cost of research re-allocation.

[17] This comes from the fact that directed technical change makes a delayed switch of research activity to the clean sector extremely costly, as explained in AABH.

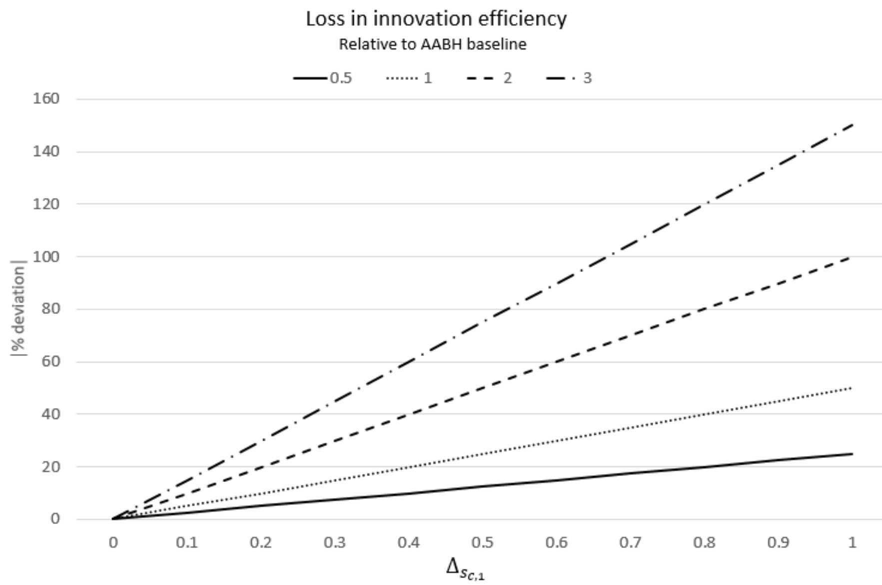

Figure 2: The expected loss in innovation efficiency relative to the frictionless AABH baseline, $-\Psi_{c,t}(\kappa)$, for different strengths of the adjustment friction, κ.

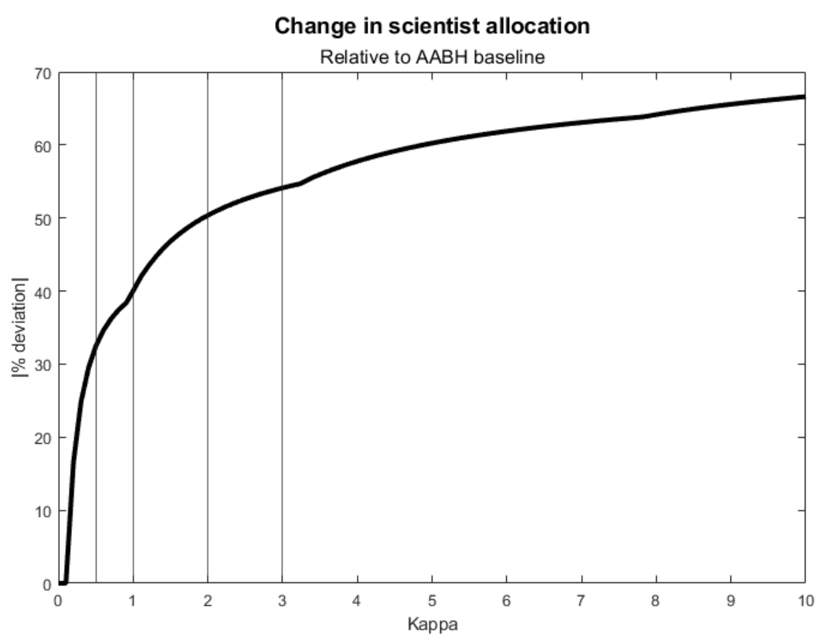

Figure 3: The first-period optimal deviation of the scientist allocation from the AABH baseline, $|s_{c,1}(\kappa) - s_{c,1}(0)|$, where $s_{c,1}(0) = 1$. The deviation is larger than 0 for $\kappa > 0.1$.

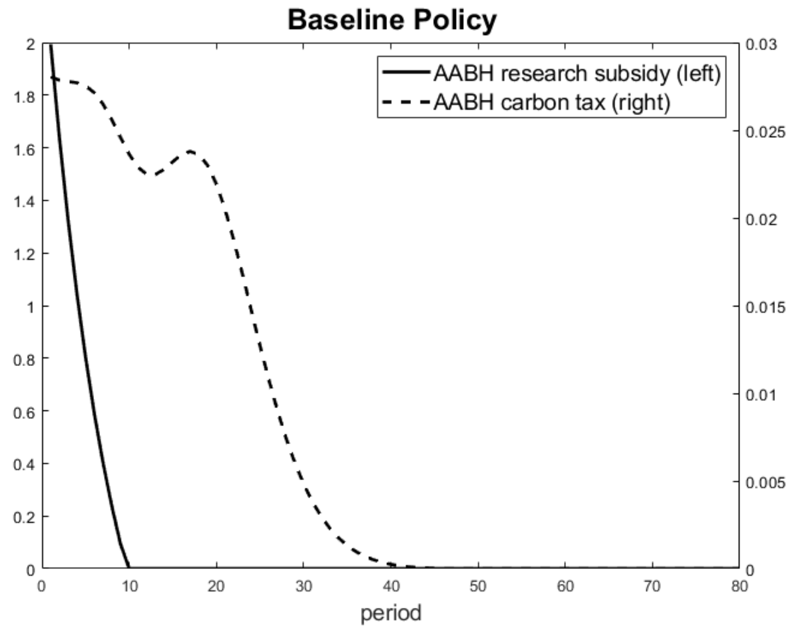

Figure 4: The baseline policy derived in AABH (κ=0) under the given parameters

baseline clean research subsidy is such that relative profits in the clean sector are *just* high enough for research activity to take place there when $\kappa = 0$, i.e. scientists are on a knife-edge. The presence of a small adjustment friction, if unaccounted for, is sufficient to lower relative profits of clean innovation to the extent that scientists fall off the green growth cliff, thereby breaking the socially optimal equilibrium in AABH, even in the presence of the optimal subsidy from AABH.

Following the initial failure of inducing clean innovation, the self-perpetuating dynamics of the model come into play. As innovation keeps taking place in the dirty sector, the dirty intermediate good keeps improving in quality, its relative price falls while its labour share rises. Because the market size effect dominates the price effect, innovation incentives in the clean sector drop further and the technology gap between the two intermediates rises. In turn, this induces final good producers to increasingly shift their input portfolio towards the dirty intermediate good. The baseline carbon tax, calibrated for the AABH social optimum where the research subsidy induces all research activity to immediately switch to the clean sector, is too delicate to induce final good producers to withstand the temptation of using the increasingly more lucrative dirty technologies in their production process — clean technologies never catch up with dirty ones and dirty technologies are locked in for the long-term.

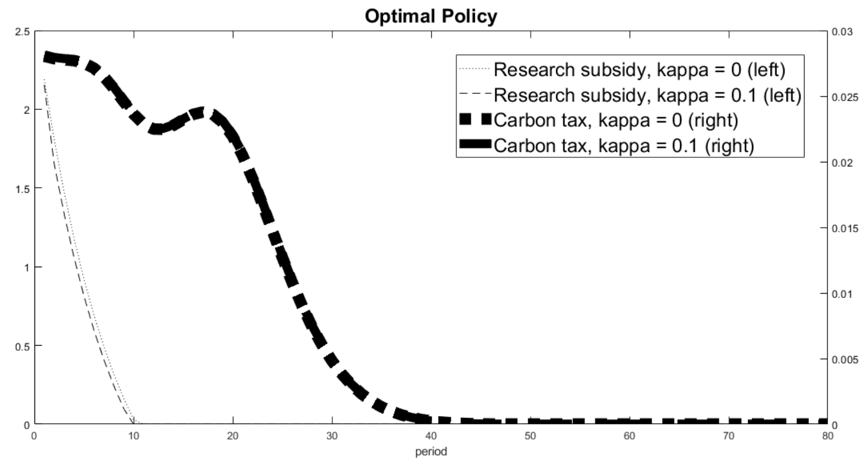

Figure 5: Optimal Policy under a very small friction ($\kappa = 0.1$ — burns only 5% of the clean innovation efficiency gains at a full switch within five years, under the ABHH baseline policy

On the one hand, this leads to higher consumption growth than in the AABH baseline economy as the final good is based on a composite to a larger extent made up of the better-developed dirty intermediate. However, because households do not internalize that their consumption relies on an exceedingly pollution-intensive composite good, the use of the dirty good silently pollutes the environment to the extent that the economy reaches an irreversible environmental disaster at which point everybody dies. Small errors in the calibration of environmental policy in this world are thus rapidly explosive.

This outcome holds a worrying message for economic policy makers. In a setting with self-perpetuating dynamics and a finite carbon budget, the risk of getting policy wrong is highly asymmetric so 'robust policy' implies erring on the side of stringency. However, note that the *optimal* policy to restore the social optimum at such a small adjustment friction in Figure 5 is very close to the AABH optimal policy. This is because, As Figure 3 shows, the socially optimal allocation under $\kappa = 0.1$ is the same as the one at the AABH baseline ($\kappa = 0$).[18] When the socially optimal allocation corresponds to the AABH baseline, the clean research subsidy rises to the occasion and compensates scientists for their loss in

[18] This equivalence result comes from the benefit of a speedy innovation reallocation outweighing the, in this case, modest deadweight loss associated with an immediate and full switch to clean innovation. Delaying the switch in innovation activity is costly with directed technical change because it increases the technology gap between clean and dirty intermediates which is increasingly costly to overcome and thus leads to a higher long-term loss in consumption.

expected profits from doing research in the clean sector so as to implement the immediate switch of all research activity to the clean sector. Because the resulting scientist allocation is identical to the one in AABH and the clean research subsidy is non-distortionary, none of the endogenous variables in (17) change and the carbon tax is identical to the one in AABH. Intuitively, because the technology gap between clean and dirty intermediates remains unchanged relative to the AABH social optimum, the financial incentive necessary to bring about the desired drop in pollution intensity of the final good relative to laissez-faire is equivalent to the carbon tax in AABH.

The central point here is that being on this knife-edge has the undesireable side effect that small degrees of oblivion on the part of the social planner are disastrous. It is hard to argue that policy makers in reality are perfectly informed of all the relevant frictions in the economy. Suppose policy makers were aware of the presence of such a friction but did not know how high κ is in their economy. Clearly, if the objective was to avoid an environmental disaster for sure, then the optimal policy corresponding to the highest reasonable κ imaginable would have to be implemented. A more pragmatic approach may be to focus less on optimal policies, i.e. the cheapest way of bringing about green growth, and more on robust policies. Spending a bit too much on supporting the economic transition may in the worst case be expensive but spending too little is for sure expensive

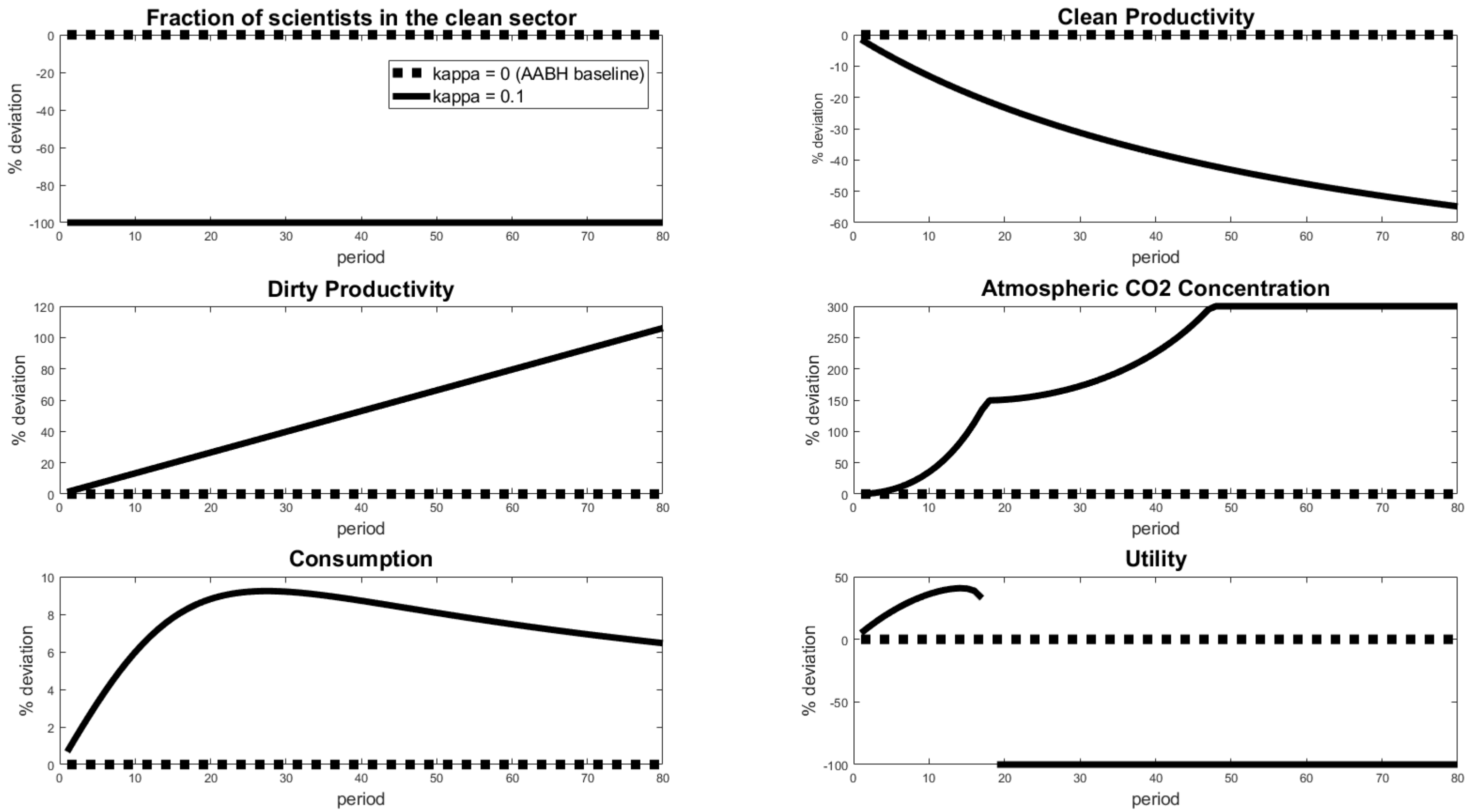

Figure 6: Model dynamics in % deviations from the AABH baseline under the *baseline* policy from AABH (κ=0) with κ=0.1

and leads to a huge loss in welfare because, as the results in this section have shown, in absence of sufficiently forceful policy action, the carbon tax is completely ineffective at preventing an environmental disaster but leads to economic distortions nonetheless.

6.3 How policy makers can turn the tide

Having shown that even a very small friction can have a large impact, I now proceed to considering the social optima under a wider range of conceivable frictions, $\kappa = \{0.5, 1, 2, 3\}$. As the top-left panel of Figure 7 shows, the socially optimal allocation under these frictions no longer corresponds to the baseline social optimum ($\kappa = 0$). Instead, innovation activity moves more gradually to the clean sector over a period of 10-30 years instead of 5 years, as in AABH. This is because the presence of the retraining friction lowers both the private and the social value of an innovation in the clean sector whenever this involves a concurrent reallocation of innovation activity to the clean sector ($\Delta_{s_{c,t}} > 0$). In (18), the presence of the friction lowers the intertemporal knowledge spillovers from clean innovation in periods where $\Delta_{s_{c,t}} > 0$.[19] This is because the friction wipes out part of the gain in clean productivity today if $\Delta_{s_{c,t}} > 0$, thereby lowering productivity spillovers in all future periods — a dynamic deadweight loss to society. This reduces the social desireability of clean research in the few but critical periods where reallocation incentives are strong.[20] It is no longer socially optimal to immediately switch all innovation activity to the clean sector as in AABH because this would entail a disproportionate current and future deadweight loss coming from the convexity of the adjustment friction. Instead, it becomes socially desireable to reallocate innovation activity more gradually over multiple periods.

A pertinent question that may arise is why the social planner does not spread out the switch of innovation activity to the clean sector across more periods. Why does the optimal slowdown of the transition to the clean sector as compared to AABH become less reactive to κ as κ rises (see Figure 3)? This goes back to a central feature of directed technical change. Everything else equal, reallocation incentives in the first periods are strong because delaying the reallocation of innovation activity to the clean sector allows the technology gap between clean and dirty technologies to grow. In the presence of a limited carbon budget, clean technologies eventually have to overcome this technology gap to prevent an environmental disaster. This longer catch-up phase later on leads to excess consumption losses because a higher carbon tax is now necessary to prevent final good producers from using the better-developed dirty input in final good production to the extent necessary for preventing an environmental disaster. Therefore, in the presence of the adjustment

[19] This is implicit in (18) because future technology is a function of current and future frictions ($A_{j,v}(\Psi_{j,v})\ \forall\ v > t, j = \{c, d\}$) but can be seen explicitly in (63) in the Appendix.

[20] As previously discussed, reallocation incentives in the first periods are strong because delaying the reallocation of innovation activity allows the technology gap between clean and dirty technologies to grow which leads to a longer catch-up phase later on with associated excess consumption losses compared to the scenario without the delay.

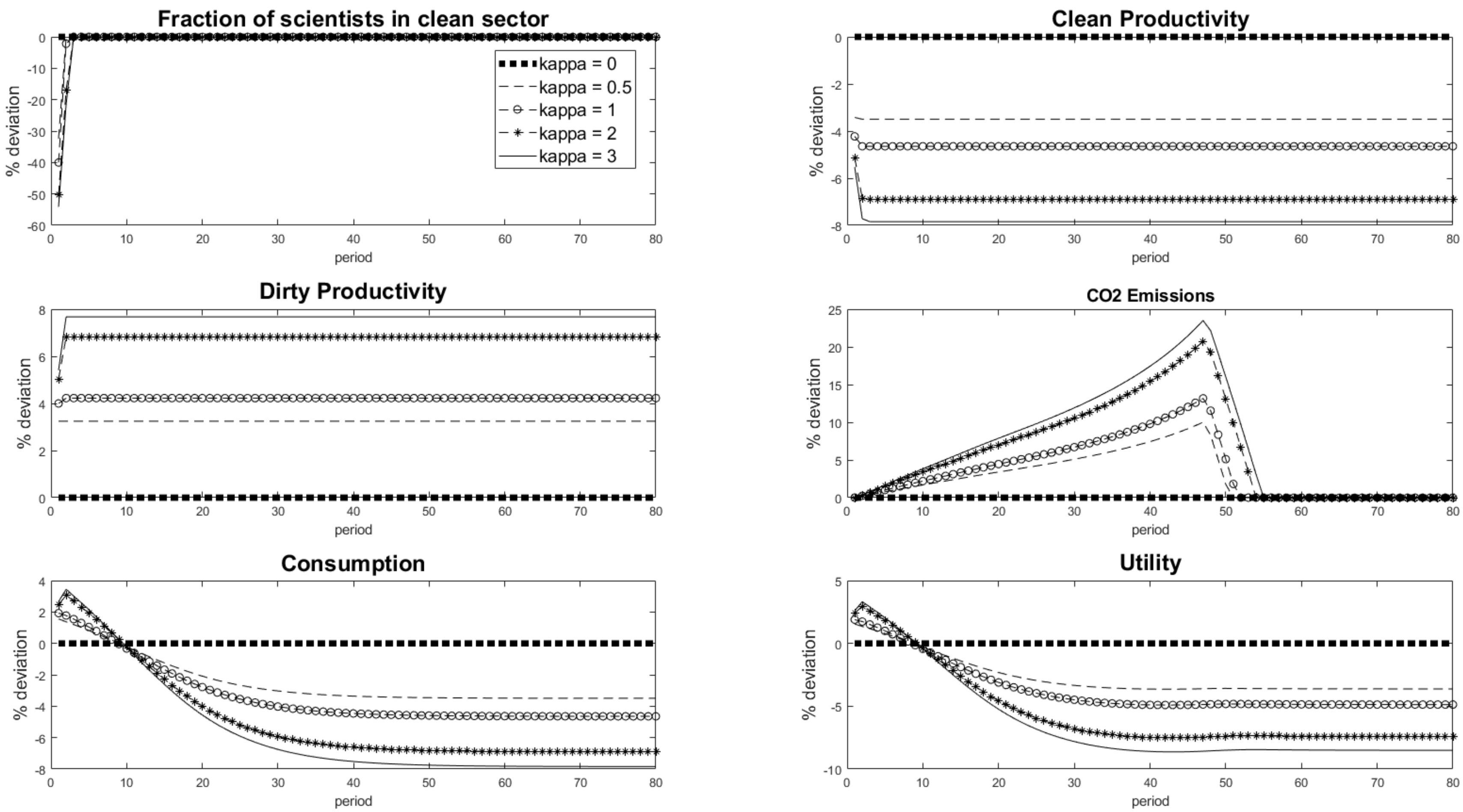

Figure 7: Model dynamics in percentage deviations relative to AABH baseline under the *optimal* policy with $\kappa = \{0, 0.5, 1, 2, 3\}$

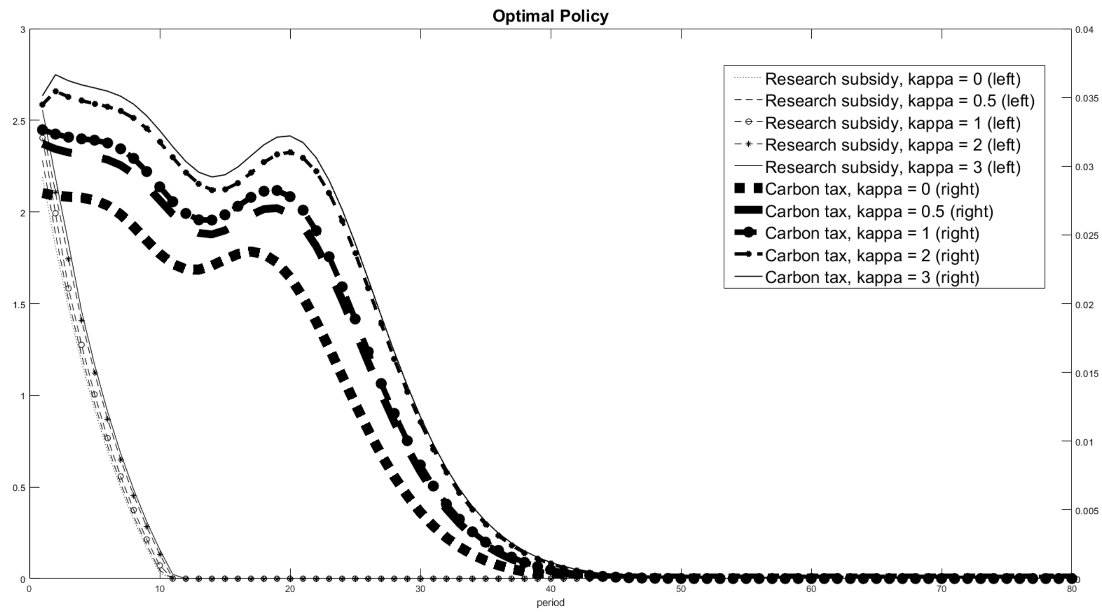

Figure 8: *Optimal* policy paths with $\kappa = \{0, 0.5, 1, 2, 3\}$

friction, there is a tension between the urge to avoid consumption losses from delaying the transition and the urge to avoid consumption losses from foregoing productivity gains due to an overly speedy transition. As a result, the optimal policies under the adjustment friction ($\kappa > 0$) in Figure 8 have a similar duration regardless of κ.

The policies required to implement the social optimum are displayed in Figure 8. A first observation is that even though optimal policy is more aggressive than the AABH baseline policy ($\kappa = 0$), it is not *that* different from it either. The clean research subsidy is stronger in the first few periods, the higher is κ. This is because it is the main purpose of the clean research subsidy to induce scientists to switch to the clean research sector even though dirty technologies are initially more productive, i.e. a more lucrative innovation opportunity. Once scientists have successfully transitioned, their innovations in the clean sector add to the level of productivity there and it takes clean technologies about 10-12 periods (50-60 years) to catch up with dirty ones at which point the clean research subsidy is no longer needed.

In the meantime, while clean technologies are catching up, final good producers prefer to use the better-developed dirty intermediate in final good production. Because the reallocation of innovation activity to the clean sector is now slower, the technology gap is larger and the carbon tax required to prevent excessive use of the dirty input therefore has to be higher. Note that the carbon tax is more aggressive in the first 30 periods (150 years) than in the AABH case but comparably less impressively longer-lived. This is because the carbon tax is crucial until the clean intermediate is sufficiently better developed than the dirty intermediate for its relative price to drop such that final good producers are not inclined to make use of it (see 12).[21] To sum up, policy close to but initially more aggressive than the AABH baseline preserves the social optimum. Simply stretching the transition out is pointless and policy needs to act bigger today to compensate for the initial losses coming from the adjustment friction.

The bottom panels of Figure 7 show that the slower transition of research activity to the clean sector under the adjustment friction leads to an intertemporal utility tradeoff. While clean productivity increases more slowly than the AABH baseline ($\kappa = 0$), the initial increase in dirty productivity, although it levels off rapidly, leads to an increasing excess deterioration of environmental quality compared to the frictionless baseline ($\kappa = 0$). As a result, the model entails an intertemporal consumption and utility tradeoff. While consumption under the stronger friction is higher in the first 50 years than in the frictionless case, it is consistently lower thereafter. This intertemporal consumption tradeoff reflects the mechanism discussed in the previous paragraph. Because clean technologies catch up slower under the friction, the final good is initially made up to a larger extent of the better-developed dirty intermediate allowing for higher consumption. Eventually, though, this technology gap has to be overcome and the larger the gap, the larger is the long-run excess consumption loss associated with it. This affects the evolution of utility. The initial benefit from higher short-term consumption paired with the eventual loss coming from the longer catch-up phase as well as the loss coming from the increasing excess deterioration of environmental quality leads to an intertemporal utility tradeoff where current generations (within the first 50 years) benefit from a slower transition at the expense of future generations (thereafter).

To sum up, under an adjustment friction, the socially optimal transition to clean technologies is slower. This should not be mistaken for a policy failure if it is accompanied by comprehensive policy action. Rather, a gradual transition to clean technologies is a socially optimal outcome in the presence of economic bottlenecks affecting the social value of the transition pace. A decisively more aggressive clean research subsidy and carbon tax *today* are essential to prevent the sluggishness of the transition from driving the economy into an environmental disaster.

[21] How long-lived and aggressive the carbon tax has to be depends on both the discount factor and the elasticity of substitution. Holding the former constant, a higher elasticity of substitutions implies that final good producers substitute the dirty intermediate for a clean version more easily and the carbon tax can be lower and shorter. However, a high elasticity of substitution is a double-edged sword under DTC because it also exacerbates carbon lock-in in absence of regulation.

7 Conclusion

Climate change is one of the most pressing economic policy challenges of the current times. In this paper, I revisited the optimal policy mix consisting of a temporary clean research subsidy and a temporary carbon tax of Acemoglu, Aghion, Bursztyn, and Hemous (2012), AABH, and investigated the effect of an adjustment friction in the research sector in order to account for the fact that the transition to green growth likely features bottlenecks. Clean research requires specific knowledge distinct from research in polluting sectors and retraining scientists to switch sectors costs time. Furthermore, new clean entrants may face entry barriers in the form of excessive capital requirements. The aggregate friction under study in this paper is consistent with both scenarios.

The paper first assesses the robustness of the optimal policy derived in AABH by assuming that the social planner is oblivious to the presence of the adjustment friction. The results show that the baseline policy in AABH is not robust to the inclusion of a small friction. Thus, if the world is not exactly as modeled in AABH, the optimal policy derived therein leads to an environmental disaster. This is because optimal policies put scientists on a knife-edge where they are only *marginally* inclined to enter the clean sector so that the smallest deterrent, such as an unaccounted friction, takes this edge away and they continue innovating in dirty technologies, leading to an environmental disaster. However, when the social planner takes the friction into account, she can prevent a disaster by implementing a substantially higher, though barely longer-lived, clean research subsidy and carbon tax.

An important policy implication here is that the risk of getting environmental policy wrong is highly asymmetric and 'robust policy' implies erring on the side of stringency. Furthermore, I show that it is then socially optimal to gradually switch all research activity to the clean sector within about 20 years — rather than 5 as in AABH. This should not be mistaken for a policy failure if it is accompanied by comprehensive policy action. Rather, a gradual transition to clean technologies is a socially optimal outcome in the presence of economic bottlenecks affecting the social value of the transition pace.

A natural extension and avenue for future work is to analyze the specific policy instruments useful for tackling such frictions. While the point of this paper was to show that adjustment frictions matter for macro outcomes regardless of the exact micro mechanism at play, quantitatively analyzing these mechanisms would require a micro-founded model. Such a framework could also offer targeted policy advice on how to best tackle this additional market failure, e.g. by suggesting suitable re-training modules for scientists in traditionally dirty fields of innovation or public guarantees that make it easier for new clean-tech firms to make the necessary capital investments to become operational.

8 Bibliography

D. Acemoglu. Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality. *The Quarterly Journal of Economics*, 113(4):1055–1089, 1998.

D. Acemoglu. Directed Technical Change. *The Review of Economic Studies*, 69:781–809, 2002. ISSN 0034-6527. doi: 10.1111/1467-937X.00226.

D. Acemoglu. Patterns of skill premia. *The Review of Economic Studies*, 70(2):199–230, 2003a. ISSN 00346527. doi: 10.1111/1467-937X.00242.

D. Acemoglu. Labor- and Capital-Augmenting Technical Change. *Journal of the European Economic Association*, 1.1:1–37, 2003b.

D. Acemoglu and F. Zilibotti. Productivity Differences. *The Quarterly Journal of Economics*, 116(2):563–606, 2001.

D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The Environment and Directed Technical Change. *The American Economic Review*, 102(1):131–166, 2012. doi: 10.1257/aer.102.1.131.

D. Acemoglu, U. Akcigit, D. Hanley, and W. Kerr. Transition to Clean Technology. *Journal of Political Economy*, 124(1), 2016. ISSN 0022-3808. doi: 10.1086/684511.

D. Acemoglu, P. Aghion, L. Barrage, and D. Hémous. Climate Change, Directed Innovation, and Energy Transition: The Long-Run Consequences of the Shale Gas Revolution. 2019.

P. Aghion, C. Hepburn, A. Teytelboym, and D. Zenghelis. Path Dependence, Innovation and the Economics of Climate Change. *The New Climate Economy Contribution Paper*, 2014. URL `http://www.cccep.ac.uk`.

K. Arrow. The Economic Implications of Learning by Doing. *The Review of Economic Studies*, 1962.

A. B. Atkinson and J. E. Stiglitz. A New View of Technological Change. *The Economic Journal*, 79(315):573–578, 1969. URL `http://www.jstor.org/stable/2230384`.

D. Consoli, G. Marin, A. Marzucchi, and F. Vona. Do green jobs differ from non-green jobs in terms of skills and human capital? *Research Policy*, 45(5):1046–1060, 6 2016. ISSN 00487333. doi: 10.1016/j.respol.2016.02.007.

D. J. Deming and K. Noray. Earnings dynamics, changing job skills, and stem careers, 11 2020. ISSN 15314650.

S. Fried. Climate policy and innovation: A quantitative macroeconomic analysis. *American Economic Journal: Macroeconomics*, 10(1):90–118, 2018. ISSN 19457715. doi: 10.1257/mac.20150289.

R. Gerlagh, S. Kverndokk, and K. E. Rosendahl. The Optimal Time Path of Clean Energy R&D Policy when Patents have Finite Lifetime. *Journal of Environmental Economics and Management*, 67(1):2–19, 1 2014. doi: 10.1016/j.jeem.2013.09.005.

M. Greaker and T.-R. Heggedal. A Comment on the Environment and Directed Technical Change. 2012. URL `http://hdl.handle.net/10419/192695www.econstor.eu`.

M. Greaker, T. R. Heggedal, and K. E. Rosendahl. Environmental Policy and the Direction of Technical Change. *Scandinavian Journal of Economics*, 120(4):1100–1138, 2018. ISSN 14679442. doi: 10.1111/sjoe.12254.

G. M. Grossman and E. Helpman. Endogenous Innovation in the Theory of Growth. *Journal of Economic Perspectives*, 8(1):23–44, 1994. ISSN 0895-3309. doi: 10.3386/w4527. URL `https://pubs.aeaweb.org/doi/pdfplus/10.1257/jep.8.1.23`.

D. Hémous. The Dynamic Impact of Unilateral Environmental Policies. *Journal of International Economics*, 103, 2016. doi: 10.1016/j.jinteco.2016.09.001.

J.-C. Hourcade, A. Pottier, and E. Espagne. The Environment and Directed Technical Change: Comment. 2012.

S. Jensen and C. Traeger. Pricing Climate Risk. 2016.

C. I. Jones and J. C. Williams. Too Much of a Good Thing? The Economics of Investment in R&D. *Journal of Economic Growth*, 5(1):65–85, 2000.

M. T. Kiley. The Supply of Skilled Labour and Skill-Biased Technological Progress Author. *The Economic Journal*, 109(458):708–724, 1999.

T. Kruse, M. Mohnen, P. Pope, and M. Sato. Green revenues, profitability and market valuation: Evidence from a global firm level dataset. 2020. ISSN 2515-5717. URL `www.cccep.ac.uk`.

D. Lemoine. Green expectations: Current effects of anticipated carbon pricing. *Review of Economics and Statistics*, 2017. ISSN 15309142. doi: 10.1162/REST{_}a{_}00627.

D. Lemoine and I. Rudik. Steering the Climate System: Using Inertia to Lower the Cost of Policy. *The American Economic Review*, 107(10):2947–2957, 2017. ISSN 00028282. doi: 10.1257/aer.20150986.

D. Lemoine and C. Traeger. Watch Your Step: Optimal Policy in a Tipping Climate. *American Economic Journal: Economic Policy*, 6(1):137–166, 2014. doi: 10.1257/pol.6J.137.

G. Marin and F. Vona. Climate policies and skill-biased employment dynamics: Evidence from EU countries. *Journal of Environmental Economics and Management*, 98, 11 2019. ISSN 10960449. doi: 10.1016/j.jeem.2019.102253.

L. Mattauch, F. Creutzig, and O. Edenhofer. Avoiding Carbon Lock-In: Policy Options for Advancing Structural Change. *Economic Modelling*, 50:49–63, 2015.

J. Noailly and R. Smeets. Directing Technical Change from Fossil-Fuel to Renewable Energy Innovation: An Application Using Firm-Level Data. *Journal of Environmental Economics and Management*, 72(C):15–37, 2015.

J. Noailly and R. Smeets. Do financing constraints matter for the direction of technical change in energy R&D? Technical report, 2019.

J. Noailly, D. Popp, and F. Vona. The Covid-19 pandemic must not undermine environmental goals. *VoxEu*, 2020.

D. Popp and R. Newell. Where does energy R & D come from ? Examining crowding out from energy R & D . *Energy Economics*, 34(4):980–991, 2012. ISSN 0140-9883. doi: 10.1016/j.eneco.2011.07.001. URL `http://dx.doi.org/10.1016/j.eneco.2011.07.001`.

F. Vona, G. Marin, D. Consoli, and D. Popp. Environmental regulation and green skills: An empirical exploration. *Journal of the Association of Environmental and Resource Economists*, 5(4):713–753, 10 2018. ISSN 23335963. doi: 10.1086/698859.

Appendix

A The green growth cliff

In this section, I visually present the knife-edge that is a central feature of the AABH equilibrium. To do so, Figure 9 shows the long-run quality of the environment as the surface height. The yellow area corresponds to cases where environmental quality rebounds and reaches a healthy state. The purple area corresponds to cases where environmental quality degrades to a point of no return. These outcomes are shown as a function of two central paramaters: (i) the strength of the adjustment friction, κ, on the right axis, and (ii), the elasticity of substitution, ε, on the left axis. In each case, the long-term quality of the environment is derived under a policy of (κ, ε) = (0,3) which corresponds to the baseline AABH model.

Figure 9 shows that the model is extremely sensitive to the elasticity of substitution, regardless of κ. As soon as ε is lower than anticipated by the social planner ($\varepsilon < 3$), the environment deteriorates to a point of no return. This is because the carbon tax implemented is insufficient to induce final good producers to substitute the dirty for the clean intermediate in production. Thus, depending on the exact value of ε, the clean research subsidy may or may not be sufficient to induce scientists to switch to the clean sector at first. But they will revert back to the dirty sector at the latest when the clean research subsidy ceases to compensate them for the sluggish uptake of their improved clean technologies by final good producers. When ε is higher than anticipated by the social planner, the healthy long-run state of the environment is preserved in absence of the adjustment friction ($\kappa = 0$) but not for most cases with the adjustment friction. This is intuitive because a higher elasticity of substitution makes the job of the social planner easier — final good producers are more willing to substitute away from the dirty intermediate good and thus the baseline carbon tax is higher than the one needed here to induce sustainable growth.

Moreover, the model is sensitive to the presence of the adjustment friction. For most of the parameter space where $\kappa > 0$, the quality of the environment deteriorates in the long-run. This is because, as explained in the main text, the adjustment friction prevents the social planner from levelling the playing field for clean and the initially better-developed dirty technologies. As a result, scientists do not switch to the clean sector, the final good is produced mainly using the polluting intermediate and the environment deteriorates. However, interestingly, there is a range for a small but positive κ and $\varepsilon > 3$, where the healthy environmental quality can be preserved. The logic is simple. A small κ leads to a small loss for scientists switching to the clean sector which is counteracted by the higher elasticity of substitution because of which there is a greater demand for new clean innovations and hence higher profits for scientists.

To sum up, Figure 9 displays the 'cliff' of the long-run quality of the environment as a function of the in the two most central parameters of this model, the elasticity of subsitution and the strength of the adjustment friction. This cliff is a result of the knife-edge case in the choice of scientists which sector to do research in implemented under the optimal AABH baseline policy.

B The Equilibrium

This Appendix provides further details and derivations of my augmented version of AABH including an adjustment friction in the research sector. I colorcode the expressions as follows:

- Black: AABH laissez-faire case

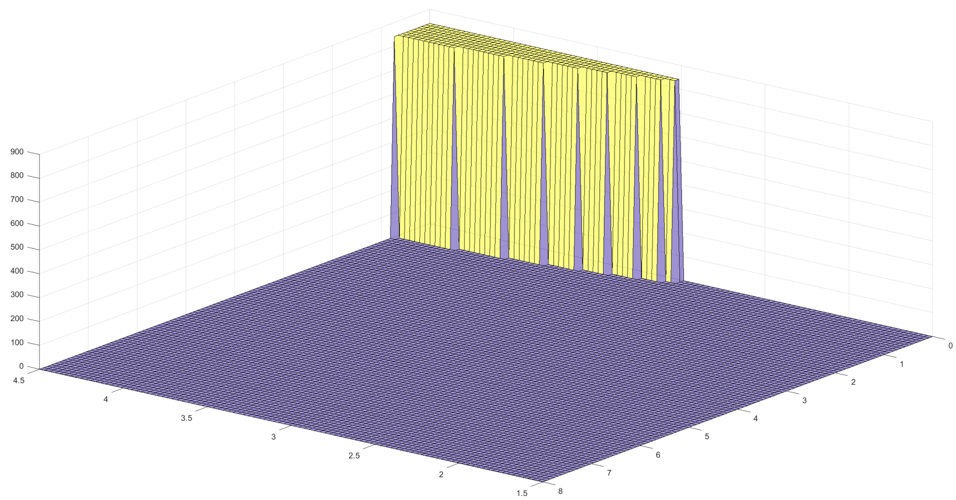

Figure 9: Long-run environmental quality (vertical axis) as a function of the strength of the adjustment friction, κ, (right axis) and the elasticity of substitution, ε, (left axis).

- Gray: Policy variables
- **Bold**: Adjustment friction in research

B.1 Optimization final good

The final good is the numéraire and thus has a price equal to 1. The final good producer maximizes:

$$\left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} - p_{c,t}Y_{c,t} - (1+\tau_t)p_{d,t}Y_{d,t}$$

Note that τ_t is the carbon tax on the use of the dirty input put in place by the social planner. The first-order conditions for the clean and dirty input respectively are:

$$Y_{c,t} = p_{c,t}^{-\varepsilon}Y_t$$
$$Y_{d,t} = [(1+\tau_t)p_{d,t}]^{-\varepsilon}Y_t$$

So that output in the clean relative to the dirty sector are an inverse function in the relative input prices:

$$\frac{Y_{c,t}}{Y_{d,t}} = \left(\frac{p_{c,t}}{(1+\tau_t)p_{d,t}}\right)^{-\varepsilon} \tag{21}$$

which corresponds to (12) in the text.

Perfect competition in the final good sector implies that firms make zero profits. Therefore:

$$0 = \underbrace{\left(\left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}}\right) + (Y_{d,t})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}}_{Y_t} - p_{c,t}Y_{c,t} - (1+\tau_t)p_{d,t}Y_{d,t}$$

Then, plugging in the previously obtained first-order conditions, I get:

$$\begin{aligned}
0 =& Y_t - p_{c,t}^{1-\varepsilon}Y_t - ((1+\tau_t)p_{d,t})^{1-\varepsilon}\, Y_t \\
1 =& p_{c,t}^{1-\varepsilon} + ((1+\tau_t)p_{d,t})^{1-\varepsilon} \\
1 =& \left[p_{c,t}^{1-\varepsilon} + (1+\tau_t)^{1-\varepsilon} p_{d,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}
\end{aligned} \tag{22}$$

B.2 Optimization intermediate good

The producer of clean or dirty intermediate goods maximizes:

$$\begin{aligned}
& p_{j,t}Y_{j,t} - w_t L_{j,t} - \int_0^1 p_{j,i,t}x_{j,i,t}di \\
=& p_{j,t}\underbrace{L_{j,t}^{1-\alpha}\int_0^1 A_{j,i,t}^{1-\alpha}x_{j,i,t}^{\alpha}di}_{Y_{j,t}} - w_t L_{j,t} - \int_0^1 p_{j,i,t}x_{j,i,t}di
\end{aligned}$$

The first-order condition with respect to capital (machines), $x_{j,i,t}$, is:

$$\begin{aligned}
p_{j,i,t} =& \alpha p_{j,t} L_{j,t}^{1-\alpha} A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha-1} \\
x_{j,i,t} =& \left(\frac{\alpha p_{j,t}}{p_{j,i,t}}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}
\end{aligned} \tag{23}$$

The first-order condition with respect to labor is:

$$w_t = (1-\alpha)\, p_{j,t} L_{j,t}^{-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha} di \tag{24}$$

B.3 Optimization machine owner

The monopolist owner of machine i in sector j chooses $p_{j,i,t}$ and $x_{j,i,t}$ to maximize profits subject to the inverse demand curve (23):

$$\begin{aligned}
\pi_{j,i,t} =& \left(1+\mathbb{I}_{\{j=c\}}\nu_{c,t}\right)\left(p_{j,i,t}-\psi(1-\chi)\right)x_{j,i,t} \\
=& \left(1+\mathbb{I}_{\{j=c\}}\nu_{c,t}\right)\left(p_{j,i,t}-\psi(1-\chi)\right)p_{j,i,t}^{-\frac{1}{1-\alpha}} \\
& (\alpha p_{j,t})^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \\
=& \underbrace{\left(1+\mathbb{I}_{\{j=c\}}\nu_{c,t}\right)(\alpha p_{j,t})^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}}_{=\xi_j} \\
& \left(p_{j,i,t}^{-\frac{\alpha}{1-\alpha}} - \psi(1-\chi)p_{j,i,t}^{-\frac{1}{1-\alpha}}\right)
\end{aligned}$$

where $\nu_{c,t}$ is the clean research subsidy and χ is the subsidy to the use of all machines put in place by the social planner, respectively. The first-order condition is:

$$\begin{aligned}
0 =& \xi_j\left(-\left(\frac{\alpha}{1-\alpha}\right)p_{j,i,t}^{-\frac{1}{1-\alpha}} + \psi(1-\chi)\left(\frac{1}{1-\alpha}\right)p_{j,i,t}^{-\frac{1}{1-\alpha}-1}\right) \\
\alpha p_{j,i,t}^{-\frac{1}{1-\alpha}} =& \psi(1-\chi)p_{j,i,t}^{-\frac{2+\alpha}{1-\alpha}} \\
p_{j,i,t} =& \left(\frac{\psi}{\alpha}\right)(1-\chi) \\
p_{j,i,t} =& \alpha(1-\chi),
\end{aligned} \tag{25}$$

where the last step follows from $\psi = \alpha^2$. Unsurprisingly, since the inverse demand curve (23) is iso-elastic, the monopolist charges a constant mark-up over his marginal costs. Plugging the result into (23) I find the equilibrium demand for machines as:

$$\begin{aligned}
x_{j,i,t} &= \left(\frac{\alpha p_{j,t}}{\alpha(1-\chi)}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \\
&= \left(\frac{p_{j,t}}{1-\chi}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t},
\end{aligned} \tag{26}$$

Demand is increasing in machine-specific productivity, the sector-specific labor force and price level (given $\alpha \in (0,1)$).

I can now obtain equilibrium profits of the machine owner in sector j by plugging (25) and (26) into her objective function:

$$
\begin{aligned}
\pi_{j,i,t} &= (1+\mathbb{I}_{\{j=c\}}\nu_{c,t})\left(p_{j,i,t}-\psi(1-\chi)\right)x_{j,i,t} \\
&= (1+\mathbb{I}_{\{j=c\}}\nu_{c,t})\left(\alpha-\alpha^2\right)(1-\chi)\left(\frac{p_{j,t}}{1-\chi}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \\
&= (1+\mathbb{I}_{\{j=c\}}\nu_{c,t})\alpha\,(1-\alpha)\,(1-\chi)^{-\frac{\alpha}{1-\alpha}} p_{j,t}^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}
\end{aligned}
$$

B.4 Equilibrium

B.4.1 Innovation

I can find the equilibrium production levels in the two sectors by combining the equilibrium demand for machines (26) with the production function of the intermediate good producers (4) for $j = \{c, d\}$, respectively.

$$
\begin{aligned}
Y_{j,t} &= L_{j,t}^{1-\alpha}\int_0^1 A_{j,i,t}^{1-\alpha}\left(\left(\frac{p_{j,t}}{1-\chi}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}\right)^{\alpha} di \\
&= \left(\frac{p_{j,t}}{1-\chi}\right)^{\frac{\alpha}{1-\alpha}} L_{j,t}A_{j,t} \qquad (27)
\end{aligned}
$$

Combining (27) for both sectors, I can find the relative equilibrium production levels:

$$
\frac{Y_{c,t}}{Y_{d,t}} = \left(\frac{p_{c,t}}{p_{d,t}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right) \qquad (28)
$$

To obtain relative profits from working in the clean versus dirty sector, I divide (14) for both sectors, I obtain:

$$
\frac{\Pi_{c,t}}{\Pi_{d,t}} = (1+\mathbb{I}_{\{j=c\}}\nu_{c,t})\boldsymbol{(1-\Psi_{c,t})}\left(\frac{\eta_c}{\eta_d}\right)\left(\frac{p_{c,t}^{\frac{1}{1-\alpha}}L_{c,t}}{p_{d,t}^{\frac{1}{1-\alpha}}L_{d,t}}\right)\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)
$$

- First, the **price effect** reflects that I prefer to innovate where I expect *higher prices* because for a given unit of innovation, I earn higher profits. For $\alpha \in (0,1)$, this is the more backward sector. To see this, I can combine (24) and (26) to obtain:

$$
\begin{aligned}
w_t =& (1-\alpha)\, p_{j,t} L_{j,t}^{-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} \left(\left(\frac{p_{j,t}}{1-\chi} \right)^{\frac{1}{1-\alpha}} L_{j,t} A_{j,i,t} \right)^{\alpha} di \\
=& (1-\alpha)\, p_{j,t}^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} \int_0^1 A_{j,i,t} di \\
=& (1-\alpha)\, p_{j,t}^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} A_{j,t} \qquad (29) \\
p_{j,t}^{\frac{1}{1-\alpha}} =& \frac{w_t}{(1-\alpha)\, A_{j,t}} (1-\chi)^{\frac{\alpha}{1-\alpha}} \\
p_{j,t} =& \left(\frac{w_t}{(1-\alpha)\, A_{j,t}} \right)^{1-\alpha} (1-\chi)^{\alpha} \qquad (30)
\end{aligned}
$$

Now combine (30) for both sectors:

$$
\frac{p_{c,t}}{p_{d,t}} = \left(\frac{A_{d,t}}{A_{c,t}} \right)^{1-\alpha} \qquad (31)
$$

Evidently, the input produced with the more productive machines will be relatively cheaper.

- Second, the **market size effect** reflects that I prefer to innovate where I expect a *larger market* (or a larger labor force) because any given unit of innovation will be met by a larger demand. To show that for $\alpha \in (0,1)$ and $\varepsilon > 1$, this is the more advanced sector, I combine (12) with (28):

$$
\begin{aligned}
\frac{p_{c,t}}{p_{d,t}} =& (1+\tau_t) \left(\frac{Y_{c,t}}{Y_{d,t}} \right)^{-\frac{1}{\varepsilon}} \\
=& (1+\tau_t) \left(\left(\frac{p_{c,t}}{p_{d,t}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_{c,t}}{L_{d,t}} \right) \left(\frac{A_{c,t}}{A_{d,t}} \right) \right)^{-\frac{1}{\varepsilon}}
\end{aligned}
$$

Now plug in from (31):

$$\frac{p_{c,t}}{p_{d,t}} =(1+\tau_t)\left(\left(\left(\left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)\right)^{-\frac{1}{\varepsilon}} \tag{32}$$

$$=(1+\tau_t)\left(\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)^{1-\alpha}\right)^{-\frac{1}{\varepsilon}} \tag{33}$$

Combining (33) with (31) yields:

$$\left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha} = (1+\tau_t)\left(\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)^{1-\alpha}\right)^{-\frac{1}{\varepsilon}}$$

$$\frac{L_{d,t}}{L_{c,t}} = (1+\tau_t)^{-\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{(1-\alpha)(1-\varepsilon)}$$

$$\frac{L_{c,t}}{L_{d,t}} = (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi} \tag{34}$$

where the last step follows from $\phi \equiv (1-\alpha)(1-\varepsilon)$. The latter simplifies to (A.5) in AABH under laissez-faire when $\tau_t = 0$.

The relatively more advanced sector attracts a relatively larger labor force and thus offers a larger market for any given innovation.

- The **direct productivity effect** reflects that I prefer to innovate where I benefit from a *higher pre-existing productivity* to 'build on the shoulders of giants'. This is by definition the more advanced sector.

B.4.2 Prices

I now derive prices in each sector as a function of productivity. First, combine (31):

$$\frac{p_{c,t}}{p_{d,t}} = \left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha}$$

and (22):

$$p_{c,t}^{1-\varepsilon} + (1+\tau_t)^{1-\varepsilon} p_{d,t}^{1-\varepsilon} = 1$$

to obtain:

$$\begin{aligned}
\left(\left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha} p_{d,t}\right)^{1-\varepsilon} + (1+\tau_t)^{1-\varepsilon} p_{d,t}^{1-\varepsilon} &= 1 \\
\left(\left(\frac{A_{d,t}}{A_{c,t}}\right)^{(1-\alpha)(1-\varepsilon)} + (1+\tau_t)^{1-\varepsilon}\right) p_{d,t}^{1-\varepsilon} &= 1 \\
\left(\left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi} + (1+\tau_t)^{1-\varepsilon}\right) p_{d,t}^{1-\varepsilon} &= 1 \\
\left(\frac{A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}}{A_{c,t}^{\phi}}\right) p_{d,t}^{1-\varepsilon} &= 1 \\
p_{d,t} = \left(\frac{A_{c,t}^{\phi}}{A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}}\right)^{\frac{1}{1-\varepsilon}} & \qquad (35)
\end{aligned}$$

Given that:

$$\frac{1-\alpha}{(1-\varepsilon)(1-\alpha)} = \frac{1}{1-\varepsilon} = \frac{1-\alpha}{\phi}$$

I can rewrite the previous expression as:

$$\begin{aligned}
p_{d,t} &= \left(A_{c,t}^{\phi}\right)^{\frac{1-\alpha}{\phi}} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}\right)^{\frac{\alpha-1}{\phi}} \\
&= A_{c,t}^{1-\alpha} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}\right)^{\frac{\alpha-1}{\phi}} \qquad (36)
\end{aligned}$$

Rearranging yields:

$$p_{d,t}^{\frac{1}{1-\alpha}} = A_{c,t} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}\right)^{-\frac{1}{\phi}} \qquad (37)$$

Now, from (31) I know that:

$$p_{c,t} = \left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha} p_{d,t}$$

Plugging in (36):

$$p_{c,t} = \left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha} A_{c,t}^{1-\alpha} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}\right)^{\frac{\alpha-1}{\phi}}$$
$$= A_{d,t}^{1-\alpha} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}\right)^{\frac{\alpha-1}{\phi}} \quad (38)$$

Rearranging yields:

$$p_{c,t}^{\frac{1}{1-\alpha}} = A_{d,t} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}\right)^{-\frac{1}{\phi}} \quad (39)$$

B.4.3 Labour

To find $L_{j,t}$, combine (34):

$$\frac{L_{c,t}}{L_{d,t}} = (1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}$$

and (5):

$$L_{c,t} + L_{d,t} = 1$$

to obtain:

$$L_{c,t} = (1+\tau_t)^{\varepsilon} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi} (1 - L_{c,t})$$
$$L_{c,t} \left(1 + (1+\tau_t)^{\varepsilon} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi}\right) = (1+\tau_t)^{\varepsilon} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi}$$
$$L_{c,t} \left(\frac{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}}{A_{c,t}^{\phi}}\right) = (1+\tau_t)^{\varepsilon} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi}$$
$$L_{c,t} = (1+\tau_t)^{\varepsilon} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi} \left(\frac{A_{c,t}^{\phi}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}}\right)$$
$$= (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}\right)^{-1} \quad (40)$$

From (5) I know that:

$$
\begin{aligned}
L_{d,t} &= 1 - L_{c,t} \\
&= 1 - (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
&= \frac{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} - (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}} \\
&= \frac{A_{c,t}^{\phi}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}} \\
&= A_{c,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1}
\end{aligned} \tag{41}
$$

B.4.4 Output

I have the equilibrium production levels from (27):

$$
Y_{j,t} = L_{j,t} A_{j,t} \left(\frac{p_{j,t}}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}}
$$

Now I can plug in for equilibrium labour, (40) and (41), and prices, (37) and (39). First, for clean output:

$$
\begin{aligned}
Y_{c,t} =& L_{c,t} A_{c,t} \left(\frac{p_{c,t}}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} A_{c,t} \\
& \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} A_{d,t}^{\alpha} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}} \\
=& \frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} A_{d,t}^{\alpha+\phi} A_{c,t} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
& \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}}
\end{aligned} \tag{42}
$$

Note that (42) simplifies under laissez-faire to (19) in AABH:

$$
\begin{aligned}
Y_{c,t} =& A_{d,t}^{\alpha+\phi} A_{c,t} \left(A_{c,t}^{\phi} + A_{d,t}^{\phi} \right)^{-1} \left(A_{d,t}^{\phi} + A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}} \\
=& A_{d,t}^{\alpha+\phi} A_{c,t} \left(A_{c,t}^{\phi} + A_{d,t}^{\phi} \right)^{-\frac{\phi+\alpha}{\phi}}
\end{aligned}
$$

Next, for dirty output:

$$
\begin{aligned}
Y_{d,t} =& L_{d,t} A_{d,t} \left(\frac{p_{d,t}}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& A_{c,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} A_{d,t} \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} \\
& A_{c,t}^{\alpha} \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}} \\
=& \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} A_{c,t}^{\alpha+\phi} A_{d,t} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
& \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}}
\end{aligned} \tag{43}
$$

Note that (43) simplifies under laissez-faire to (19) in AABH:

$$
\begin{aligned}
Y_{d,t} =& A_{c,t}^{\alpha+\phi} A_{d,t} \left(A_{c,t}^{\phi} + A_{d,t}^{\phi} \right)^{-1} \left(A_{d,t}^{\phi} + A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}} \\
=& A_{c,t}^{\phi+\alpha} A_{d,t} \left(A_{d,t}^{\phi} + A_{c,t}^{\phi} \right)^{-\frac{\alpha+\phi}{\phi}}
\end{aligned}
$$

To find the equilibrium total production level, express $Y_{c,t}$ as a function of $Y_{d,t}$ starting from (27):

$$
Y_{c,t} = \left(\frac{p_{c,t}}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} L_{c,t} A_{c,t}
$$

and then plugging in (34)

$$
L_{c,t} = (1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-\phi} L_{d,t}
$$

and (31)

$$
p_{c,t} = \left(\frac{A_{d,t}}{A_{c,t}} \right)^{1-\alpha} p_{d,t}
$$

to obtain:

$$Y_{c,t} = \left(\left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha} \frac{p_{d,t}}{1-\chi}\right)^{\frac{\alpha}{1-\alpha}} (1+\tau_t)^{\varepsilon} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\phi} L_{d,t} A_{c,t}$$
$$= \frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\alpha+\phi} A_{c,t} L_{d,t} p_{d,t}^{\frac{\alpha}{1-\alpha}}$$
$$= \frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\alpha+\phi} \left(\frac{A_{c,t}}{A_{d,t}}\right) \underbrace{A_{d,t} L_{d,t} p_{d,t}^{\frac{\alpha}{1-\alpha}}}_{Y_{d,t}}$$
$$= \frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\alpha+\phi-1} Y_{d,t}$$

Now, plug the latter into (3):

$$Y_t = \left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$= \left(\left(\frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\alpha+\phi-1} Y_{d,t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$= \left(Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}} \left(\left(\frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\alpha+\phi-1}\right)^{\frac{\varepsilon-1}{\varepsilon}} + 1\right)\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$= Y_{d,t} \left(\left(\frac{(1+\tau_t)^{\varepsilon}}{(1-\chi)^{\frac{\alpha}{1-\alpha}}} \left(\frac{A_{d,t}}{A_{c,t}}\right)^{\alpha+\phi-1}\right)^{\frac{\varepsilon-1}{\varepsilon}} + 1\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Now remember that

$$\phi = (1-\alpha)(1-\varepsilon)$$
$$1-\varepsilon = \frac{\phi}{1-\alpha}$$
$$\varepsilon-1 = \frac{\phi}{\alpha-1}$$
$$\varepsilon = \frac{\phi+\alpha-1}{\alpha-1}$$
$$\frac{\varepsilon-1}{\varepsilon} = \frac{\phi}{\phi+\alpha-1}$$

Plugging this into the previous expression, I get:

$$
\begin{aligned}
Y_t =& Y_{d,t} \left(\frac{(1+\tau_t)^{\frac{\phi}{\alpha-1}}}{(1-\chi)^{\frac{\alpha\phi}{(1-\alpha)(\phi+\alpha-1)}}} \left(\frac{A_{d,t}}{A_{c,t}} \right)^{\phi} + 1 \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
=& Y_{d,t} \left(\frac{(1+\tau_t)^{\varepsilon-1}(1-\chi)^{\frac{\alpha\phi}{(1-\alpha)(1-\alpha-\phi)}} A_{d,t}^{\phi} + A_{c,t}^{\phi}}{A_{c,t}^{\phi}} \right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}
$$

Plugging in for the exponent and dirty output from (43), I obtain:

$$
\begin{aligned}
Y_t =& \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} A_{c,t}^{\alpha+\phi} A_{d,t} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
& \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}} \\
& \left(\frac{(1+\tau_t)^{\varepsilon-1}(1-\chi)^{\frac{\alpha\phi}{(1-\alpha)(1-\alpha-\phi)}} A_{d,t}^{\phi} + A_{c,t}^{\phi}}{A_{c,t}^{\phi}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
=& \left(\frac{1}{1-\chi} \right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
& \left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} \right)^{-\frac{\alpha}{\phi}} \\
& \left((1+\tau_t)^{\frac{\phi}{\alpha-1}} (1-\chi)^{\frac{\alpha\phi}{(1-\alpha)(1-\alpha-\phi)}} A_{d,t}^{\phi} + A_{c,t}^{\phi} \right)^{\frac{\phi+\alpha-1}{\phi}}
\end{aligned}
\tag{44}
$$

Note that (44) simplifies to (19) in AABH in the laissez-faire case:

$$
\begin{aligned}
Y_t =& A_{d,t} A_{c,t}^{\phi+\alpha} \left(A_{d,t}^{\phi} + A_{c,t}^{\phi} \right)^{-\frac{\alpha+\phi}{\phi}} \\
& \left(\left(A_{d,t}^{\phi} + A_{c,t}^{\phi} \right) A_{c,t}^{-\phi} \right)^{\frac{\phi+\alpha-1}{\phi}} \\
=& A_{d,t} A_{c,t} \left(A_{d,t}^{\phi} + A_{c,t}^{\phi} \right)^{-\frac{1}{\phi}}
\end{aligned}
$$

B.4.5 Scientist allocation

I divide (14) for both sectors:

$$\frac{\Pi_{c,t}}{\Pi_{d,t}} = (1+\nu_{c,t})\mathbf{(1-\Psi_{c,t})}\left(\frac{\eta_c}{\eta_d}\right)\left(\frac{p_{c,t}}{p_{d,t}}\right)^{\frac{1}{1-\alpha}}$$
$$\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)$$

with (34) and (31) to obtain:

$$\begin{aligned}\frac{\Pi_{c,t}}{\Pi_{d,t}} =& (1+\nu_{c,t})\mathbf{(1-\Psi_{c,t})}\left(\frac{\eta_c}{\eta_d}\right)\left(\left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \\ & (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right) \\ =& (1+\nu_{c,t})(1+\tau_t)^{\varepsilon}\mathbf{(1-\Psi_{c,t})}\left(\frac{\eta_c}{\eta_d}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-(1+\phi)} \\ & \left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)\end{aligned}$$

Plugging in the law of motion of technology from (10):

$$A_{j,t} = (1+\mathbf{(1-\Psi_{j,t})})\,\gamma\eta_j s_{j,t} A_{j,t-1}$$

I obtain:

$$\begin{aligned}\frac{\Pi_{c,t}}{\Pi_{d,t}} =& (1+\nu_{c,t})(1+\tau_t)^{\varepsilon}\mathbf{(1-\Psi_{c,t})}\left(\frac{\eta_c}{\eta_d}\right) \\ & \left(\frac{(1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t})\,A_{c,t-1}}{(1+\gamma\eta_d s_{d,t})\,A_{d,t-1}}\right)^{-(1+\phi)} \\ & \left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)\end{aligned}$$

Note that since $s_{c,t} = 1 - s_{d,t}\ \forall t$, we have that $\Delta_{s_{dt}} = s_{d,t} - s_{d,t-1} = (1 - s_{c,t}) - (1 - s_{c,t-1}) = -(s_{c,t} - s_{c,t-1}) = -\Delta_{s_{ct}}$ the former can be rewritten as:

$$\frac{\Pi_{c,t}}{\Pi_{d,t}} = (1+\nu_{c,t})(1+\tau_t)^{\varepsilon}\mathbf{(1-\Psi_{c,t})}\left(\frac{\eta_c}{\eta_d}\right)\left(\frac{1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}{1+\gamma\eta_d(1-s_{c,t})}\right)^{-(1+\phi)}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)^{-\phi}$$

This corresponds to (16) in the text.

As in AABH, I can write (16) as a function of the decision variable, $\frac{\Pi_{c,t}}{\Pi_{d,t}} = f(s_{c,t})$. It follows that if $f(1) > 1$, i.e. if profits are relatively higher in the clean sector when the scientist chooses to engage there, then $s_{c,t} = 1$ is an equilibrium. If $f(0) < 1$, i.e. if profits are relatively higher in the dirty sector when the scientist chooses to engage there, then $s_{c,t} = 0$ is an equilibrium. If $f(s^*_{c,t}) = 1$ for some $s^*_{c,t} \in (0,1)$, i.e. if there is an allocation such that research is being done in both sectors and the scientist is indifferent between engaging in the clean or dirty sector, then $s_{c,t} = s^*_{c,t}$ is an equilibrium. These observations give putative local equilibria but they are not informative on whether, given the shape of the relative profit function in $s_{c,t}$, these equilibria are unique. To assess this, the shape of the function $f(s_{c,t})$ is checked numerically each period and the appropriate solution routine among the following is chosen:

1. If f(s) is strictly decreasing or strictly increasing in s or if f(s) is a constant, then 1)-3) in the proof of Lemma I in Appendix B of AABH apply.

2. If f(s) is u-shaped, then

   - if $f(0) < 1$ and $f(1) < 1$, then $s = 0$ is the unique equilibrium.
   - if $f(0) < 1$ and $f(1) > 1$, then there are three equilibria, an interior one $s = s^* \in (0,1)$ where s^* is such that $f(s^*) = 1$, $s = 0$ and $s = 1$.
   - if $f(0) > 1$ and $f(1) < 1$, then there is a unique interior equilibrium $s = s^* \in (0,1)$ where s^* is such that $f(s^*) = 1$.
   - if $f(0) > 1$ and $f^{\min} < 1$ and $f(1) > 1$, then there are three equilibria, two interior ones $s = \{\underline{s}^*, \overline{s}^*\} \in (0,1)$ where both allocations are such that $f(\underline{s}^*) = 1$ and $f(\overline{s}^*) = 1$, respectively, and $s = 1$.
   - if $f(0) > 1$ and $f^{\min} > 1$ and $f(1) > 1$, then $s = 1$ is the unique equilibrium.

3. If f(s) is inverse-u shaped, there can be multiple equilibria. Specifically,

   - if $f(0) < 1$ and $f^{\max} < 1$ and $f(1) < 1$, then $s = 0$ is the unique equilibrium.
   - if $f(0) < 1$ and $f^{\max} > 1$ and $f(1) < 1$, then there are three equilibria, two interior ones $s = \{\underline{s}^*, \overline{s}^*\} \in (0,1)$ where both allocations are such that $f(\underline{s}^*) = 1$ and $f(\overline{s}^*) = 1$, respectively, and $s = 0$.
   - if $f(0) < 1$ and $f(1) > 1$, then there are three equilibria, an interior one $s = s^* \in (0,1)$ where s^* is such that $f(s^*) = 1$, $s = 0$ and $s = 1$.
   - if $f(0) > 1$ and $f(1) < 1$, then there is a unique interior equilibrium $s = s^* \in (0,1)$ where s^* is such that $f(s^*) = 1$.
   - if $f(0) > 1$ and $f(1) > 1$, then $s = 1$ is the unique equilibrium.

B.4.6 Assumption I

Assumption 1 *The relative initial level of productivity in the clean versus dirty sector is such that, in absence of policy intervention, scientists will start innovating in the dirty sector.*

$$
\begin{aligned}
\frac{A_{c,0}}{A_{d,0}} < \min \Bigg\{ & \left(\frac{1 - \mathbb{N}^{+}_{s_{c,t}} \left(\frac{\kappa}{2}\right) (1 - s_{c,0})^2}{1 + \mathbb{N}^{+}_{s_{d,t}} \left(\frac{\kappa}{2}\right) (1 - s_{c,0})^2} \right)^{\frac{1}{\phi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\phi}} \left(1 + \left(1 - \mathbb{N}^{+}_{s_{c,t}} \left(\frac{\kappa}{2}\right) (1 - s_{c,0})^2 \right) \gamma \eta_c \right)^{-\frac{(1+\phi)}{\phi}}, \\
& \left(\frac{1 - \mathbb{N}^{+}_{s_{c,t}} \left(\frac{\kappa}{2}\right) s_{c,0}^2}{1 + \mathbb{N}^{+}_{s_{d,t}} \left(\frac{\kappa}{2}\right) s_{c,0}^2} \right)^{\frac{1}{\phi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\phi}} \left(1 + \left(1 + \mathbb{N}^{+}_{s_{d,t}} \left(\frac{\kappa}{2}\right) s_{c,0}^2 \right) \gamma \eta_d \right)^{\frac{(1+\phi)}{\phi}}, \\
& \left(\frac{1 - \mathbb{N}^{+}_{s_{c,t}} \left(\frac{\kappa}{2}\right) (s^{max} - s_{c,0})^2}{1 + \mathbb{N}^{+}_{s_{d,t}} \left(\frac{\kappa}{2}\right) (s^{max} - s_{c,0})^2} \right)^{\frac{1}{\phi}} \left(\frac{\eta_c}{\eta_d} \right)^{\frac{1}{\phi}} \\
& \left(\frac{1 + \left(1 - \mathbb{N}^{+}_{s_{c,t}} \left(\frac{\kappa}{2}\right) (s^{max} - s_{c,0})^2 \right) \gamma \eta_c s^{max}}{1 + \left(1 + \mathbb{N}^{+}_{s_{d,t}} \left(\frac{\kappa}{2}\right) (s^{max} - s_{c,0})^2 \right) \gamma \eta_d (1 - s^{max})} \right)^{-\frac{(1+\phi)}{\phi}} \quad \forall s^{max} \in [0, 1] \Bigg\}
\end{aligned}
$$

where $s_{c,0}$ *is the scientist allocation in period zero and* s^{max} *is the the scientist allocation maximizing the relative profit function in (16).*

The expression in Assumption 1 is based on the relative expected profit function in (16). To get a constraint on the initial conditions ensuring that, in laissez faire, the expected profits in clean research are smaller than those in dirty research, set expected relative profits equal to one, solve for relative technology levels and impose that they be smaller than relative expected dirty profits — regardless of the scientist allocation, i.e. there is no scientist allocation for which research in the clean sector is worthwhile in period one. Because of the self-perpetuating dynamics of the model, this limit on the initial conditions suffices to ensure that, in laissez-faire, there will *never* be research in the clean sector.

Specifically, the first (second) expression in the curly brackets on the right-hand side corresponds to the expected profits of doing research in the dirty versus in the clean sector for a scientist who chooses to engage in the clean (dirty) sector if the relative technologies were equally developed. To fully rule out the possibility of clean-sector research under laissez-faire, the third expression serves to rule out multiple equilibria which may arise in the case of the relative profit function being inverse-u shaped, a possibility induced by the presence of the adjustment friction (more on this later). In this case, relative clean productivity initially has to be lower than one, i.e. relative dirty profits under any of the two possible interior profit-maximizing scientist allocations. Intuitively, the criterion that relative clean productivity initially has to be smaller than relative dirty profits reflects that the force of inertia of relative productivity (path dependence) must be such that scientists are drawn to the dirty sector in the first period. The min function ensures that whichever of these three constraints is the strictest will be satisfied. Note that the presence of the friction makes the permissible technology gap consistent with pure dirty research incentives *smaller*, i.e. green technologies can be relatively *more* advanced than in AABH because for any given level of relative technologies, a stronger friction discourages scientists from switching to clean technologies.[22]

C The Social Planner Problem

$$\max_{\{C_t,S_t\}} \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \tag{45}$$

$$s.t.\; Y_t = \left(\sum_{j=c,d} Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{46}$$

$$Y_{j,t} = L_{j,t}^{1-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha} di,\; j = \{c,d\} \tag{47}$$

$$C_t + \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) = Y_t \tag{48}$$

$$L_{c,t} + L_{d,t} \leq 1 \tag{49}$$

$$s_{c,t} + s_{d,t} \leq 1 \tag{50}$$

$$A_{j,t} = (1 + \eta_j \boldsymbol{(1 - \Psi_{j,t})}_{j,t})\, A_{j,t-1},\; j = \{c,d\} \tag{51}$$

$$S_{t+1} = \min\{\max[-\xi Y_{d,t} + (1+\delta)\, S_t; 0]; \bar{S}\} \tag{52}$$

The resulting Lagrangian is:

[22] To see this, set $s_{c,0} = 0$ and notice that $\phi < 0$ when $\varepsilon > 1$. Then, the green terms in the first and third expression of Assumption 1 are all increasing in κ, the strength of the friction, i.e. clean technologies can be *more developed* and still not be attractive for scientists in the case with the friction as compared to AABH. Under the same parameter restrictions, the second expression is independent of the friction.

$$\mathbb{L} = \sum_{t=1}^{\infty} \Bigg\{ \frac{1}{(1+\rho)^t} u(C_t, S_t) - \lambda_t \left(Y_t - \left(\sum_{j=c,d} Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right)$$

$$- \sum_{j=c,d} \lambda_{j,t} \left(Y_{j,t} - (L_{j,t})^{1-\alpha} \int_0^1 (A_{j,i,t})^{1-\alpha} (x_{j,i,t})^{\alpha} \, di \right)$$

$$-\gamma_t \left(-C_t - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) + Y_t \right)$$

$$-\omega_{t+1} \left(S_{t+1} - \min\{\max[-\xi Y_{d,t} + (1+\delta) S_t; 0]; \bar{S}\} \right)$$

$$- \sum_{j=c,d} \mu_{j,,t} \left(A_{j,t} - (1 + \eta_j \boldsymbol{(1 - \Psi_{j,t})} \gamma s_{j,t}) A_{j,t-1} \right) \Bigg\}$$

C.1 The optimal carbon tax

$$\frac{\partial \mathbb{L}}{\partial Y_t} = -\lambda_t - \gamma_t = 0$$

$$\frac{\partial \mathbb{L}}{\partial C_t} = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_{t+1})}{\partial C_t} + \gamma_t = 0$$

$$\Longrightarrow \lambda_t = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial C_t} \tag{53}$$

$$\frac{\partial \mathbb{L}}{\partial S_t} = \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial S_t} + \mathbf{I}_{\{S_t < \overline{S}\}} (1+\delta) \omega_{t+1} - \omega_t = 0$$

$$= \frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial S_t}$$

$$+ \mathbf{I}_{\{S_t < \overline{S}\}} (1+\delta) \left(\frac{1}{(1+\rho)^{t+1}} \frac{\partial u(C_{t+1}, S_{t+1})}{\partial S_{t+1}} \right.$$

$$\left. + (1+\delta) \mathbf{I}_{\{S_{t+1} < \overline{S}\}} \omega_{t+2} \right) = \omega_t$$

$$= \ldots$$

$$= \sum_{j=t}^{\infty} \frac{1}{(1+\rho)^j} \frac{\partial u(C_j, S_j)}{\partial S_j} (1+\delta)^{j-t} \mathbf{I}_{\{S_j < \overline{S}\}} = \omega_t \tag{54}$$

where

$$\mathbf{I}_{\{S_t < \overline{S}\}} = \begin{cases} 1 & \text{if } S_t < \overline{S} \\ 0 & \text{otherwise} \end{cases}$$

$$
\begin{aligned}
\frac{\partial \mathbb{L}}{\partial Y_{c,t}} =& \lambda_t \left(\frac{\varepsilon}{\varepsilon-1}\right) \left((Y_{c,t})^{\frac{\varepsilon-1}{\varepsilon}} + (Y_{d,t})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}-1} \left(\frac{\varepsilon-1}{\varepsilon}\right) Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}-1} \\
&- \lambda_{c,t} = 0 \\
=& \left((Y_{c,t})^{\frac{\varepsilon-1}{\varepsilon}} + (Y_{d,t})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} Y_{c,t}^{-\frac{1}{\varepsilon}} = \frac{\lambda_{c,t}}{\lambda_t} \equiv \hat{p}_{c,t} \qquad (55)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathbb{L}}{\partial Y_{d,t}} =& \lambda_t \left(\frac{\varepsilon}{\varepsilon-1}\right) \left((Y_{c,t})^{\frac{\varepsilon-1}{\varepsilon}} + (Y_{d,t})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}-1} \left(\frac{\varepsilon-1}{\varepsilon}\right) Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}-1} \\
&- \lambda_{d,t} - \omega_{t+1}\xi = 0 \\
=& \left((Y_{c,t})^{\frac{\varepsilon-1}{\varepsilon}} + (Y_{d,t})^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} Y_{d,t}^{-\frac{1}{\varepsilon}} - \frac{\omega_{t+1}\xi}{\lambda_t} = \frac{\lambda_{d,t}}{\lambda_t} \equiv \hat{p}_{d,t} \qquad (56)
\end{aligned}
$$

The wedge in the latter expression represents the optimal carbon tax:

$$
\tau_t^* = \frac{\omega_{t+1}\xi}{\lambda_t \hat{p}_{dt}} \qquad (57)
$$

To find the equilibrium expression (adjusted equation 23 in AABH), combine the latter with (53) and (54):

$$
\begin{aligned}
\tau_t^* &= \left(\frac{\xi}{\hat{p}_{d,t}}\right) \frac{\sum_{j=t+1}^{\infty} \frac{1}{(1+\rho)^j} \frac{\partial u(C_j,S_j)}{\partial S_j} (1+\delta)^{j-(t+1)} \mathbf{I}_{\{S_j<\overline{S}\}}}{\frac{1}{(1+\rho)^t} \frac{\partial u(C_t,S_t)}{\partial C_t}} \\
&= \left(\frac{\xi}{\hat{p}_{d,t}}\right) \frac{\frac{1}{(1+\rho)} \sum_{j=t+1}^{\infty} \left(\frac{1+\delta}{1+\rho}\right)^{j-(t+1)} \frac{\partial u(C_j,S_j)}{\partial S_j} \mathbf{I}_{\{S_j<\overline{S}\}}}{\frac{\partial u(C_t,S_t)}{\partial C_t}}
\end{aligned}
$$

This corresponds to (17) in the text.

C.2 The optimal subsidy to the use of *all* machines

$$
\begin{aligned}
\frac{\partial \mathbb{L}}{\partial x_{j,i,t}} &= \lambda_{j,t} \alpha L_{j,t}^{1-\alpha} A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha-1} + \gamma_t \psi = 0 \\
\implies x_{j,i,t}^{1-\alpha} &= \frac{\lambda_{j,t}\alpha}{\gamma_t \psi} (L_{j,t} A_{j,i,t})^{1-\alpha} \\
x_{j,i,t} &= \left(\frac{\lambda_{j,t}\alpha}{\lambda_t \psi}\right)^{\frac{1}{1-\alpha}} L_{j,t} A_{j,i,t} \\
x_{j,i,t} &= \left(\frac{\alpha}{\psi} \hat{p}_{j,t}\right)^{\frac{1}{1-\alpha}} L_{j,t} A_{j,i,t} \qquad (58)
\end{aligned}
$$

Remembering that $p_{j,i,t} = \frac{\psi}{\alpha}$, the latter becomes:

$$x_{j,i,t} = \left(\frac{\hat{p}_{j,t}}{p_{j,i,t}}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \tag{59}$$

Comparing this with the *laissez-faire* equilibrium demand for machines:

$$x_{j,i,t} = \left(\frac{\alpha p_{j,t}}{p_{j,i,t}}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \tag{60}$$

shows that, given prices, machines will be used more intensely in the socially optimal allocation since $\alpha < 1$. How can the government implement the socially optimal allocation? Clearly, it has to subsidize machines by $\chi = (1-\alpha)$ so that intermediate goods producer only have to pay a fraction α of the price, $p_{j,i,t}$.

By plugging the socially optimal demand for machines (59), that is, demand in presence of the subsidy to all machines, into the intermediate good production function (47), I obtain:

$$\begin{aligned}
Y_{j,t} =& L_{j,t}^{1-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha} di \\
=& L_{j,t}^{1-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} \left(\left(\frac{\hat{p_{j,t}}}{p_{j,i,t}}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}\right)^{\alpha} di \\
=& L_{j,t}^{1-\alpha} A_{j,t}^{1-\alpha} \left(\left(\frac{\hat{p_{j,t}}}{\frac{\psi}{\alpha}}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,t}\right)^{\alpha} \\
=& L_{j,t}A_{j,t} \left(\frac{\alpha \hat{p_{j,t}}}{\psi}\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{61}$$

and given that $\psi = \alpha^2$:

$$Y_{j,t} = L_{j,t}A_{j,t} \left(\frac{\hat{p_{j,t}}}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \tag{62}$$

I can furthermore see that production is scaled up by a factor of $\alpha^{-\frac{\alpha}{1-\alpha}}$ compared to the laissez-faire level of output, $Y_{j,t} = L_{j,t}A_{j,t}p_{j,t}^{\frac{\alpha}{1-\alpha}}$. This is a consequence of the presence of the monopoly distortion in the laissez-faire equilibrium.

C.3 The optimal clean research subsidy

I now compute the optimal clean research subsidy in absence of the monopoly distortion.

$$\frac{\partial \mathbb{L}}{\partial A_{j,t}} = \lambda_{j,t}(1-\alpha)L_{j,t}^{1-\alpha}A_{j,i,t}^{-\alpha}x_{j,i,t}^{\alpha} - \mu_{j,t} + \mu_{j,t+1}\left(1 + \boldsymbol{(1-\Psi_{j,t})}\gamma\eta_j s_{j,t}\right)$$

Plugging in for $x_{j,i,t}$ from (59):

$$\begin{aligned}\frac{\partial \mathbb{L}}{\partial A_{j,t}} =& \lambda_{j,t}(1-\alpha)L_{j,t}^{1-\alpha}A_{j,i,t}^{-\alpha}\left(\left(\frac{\hat{p}_{j,t}}{p_{j,i,t}}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}\right)^{\alpha} \\ &- \mu_{j,t} + \mu_{j,t+1}\left(1 + \boldsymbol{(1-\Psi_{j,t})}\gamma\eta_j s_{j,t}\right) = 0 \\ \Longrightarrow \mu_{j,t} =& \lambda_{j,t}(1-\alpha)L_{j,t}\left(\frac{\hat{p}_{j,t}}{p_{j,i,t}}\right)^{\frac{\alpha}{1-\alpha}} + \mu_{j,t+1}\left(1 + \boldsymbol{(1-\Psi_{j,t})}\gamma\eta_j s_{j,t}\right)\end{aligned}$$

Now I plug in $p_{j,i,t} = \frac{\psi}{\alpha}$ and $\lambda_{j,t} = \hat{p}_{j,t}\lambda_t$ to obtain:

$$\begin{aligned}\underbrace{\mu_{j,t}}_{\substack{\text{Shadow price of a unit increase}\\ \text{in avg. prod. in sector j}}} =& \underbrace{\lambda_t\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_{j,t}\hat{p}_{j,t}^{\frac{1}{1-\alpha}}}_{\text{Marginal contribution to current utility}} + \\ & \underbrace{\mu_{j,t+1}\left(1 + \boldsymbol{(1-\Psi_{j,t})}\gamma\eta_j s_{j,t}\right)}_{\text{Intertemporal knowledge externality}}\end{aligned} \tag{63}$$

which is equivalent to (A.13) in AABH when I set $\kappa = \epsilon = 0$. Iterating forward, I obtain:

$$
\begin{aligned}
\mu_{j,t} =& \lambda_t \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) L_{j,t} \hat{p}_{j,t}^{\frac{1}{1-\alpha}} + (1 + \mathbf{(1-\Psi_{j,t+1})} \gamma \eta_j s_{j,t+1}) \\
& \left(\lambda_{t+1} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) L_{j,t+1} \hat{p}_{j,t+1}^{\frac{1}{1-\alpha}} + \mu_{j,t+2} \left(1 + \mathbf{(1-\Psi_{j,t+2})} \right.\right. \\
& \left.\left. \gamma \eta_j s_{j,t+2} \right)\right) \\
=& \lambda_t \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) L_{j,t} \hat{p}_{j,t}^{\frac{1}{1-\alpha}} + (1 + \mathbf{(1-\Psi_{j,t+1})} \gamma \eta_j s_{j,t+1}) \\
& \left(\lambda_{t+1} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) L_{j,t+1} \hat{p}_{j,t+1}^{\frac{1}{1-\alpha}} + (1 + \mathbf{(1-\Psi_{j,t+2})} \gamma \eta_j s_{j,t+2}) \right. \\
& \left(\lambda_{t+2} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) L_{j,t+2} \hat{p}_{j,t+2}^{\frac{1}{1-\alpha}} + (1 + \mathbf{(1-\Psi_{j,t+3})} \gamma \eta_j s_{j,t+3}) \right. \\
& \left.\left. \mu_{j,t+3} \right)\right)
\end{aligned}
$$

Remember that:

$$
\begin{aligned}
A_{j,t} =& (1 + \mathbf{(1-\Psi_{j,t})} \gamma \eta_j s_{j,t}) A_{j,t-1} \\
=& (1 + \mathbf{(1-\Psi_{j,t})} \gamma \eta_j s_{j,t}) (1 + \mathbf{(1-\Psi_{j,t-1})} \gamma \eta_j s_{j,t-1}) A_{j,t-2} \\
=& (1 + \mathbf{(1-\Psi_{j,t})} \gamma \eta_j s_{j,t}) (1 + \mathbf{(1-\Psi_{j,t-1})} \gamma \eta_j s_{j,t-1}) \\
& (1 + \mathbf{(1-\Psi_{j,t-2})} \gamma \eta_j s_{j,t-2}) A_{j,t-3} \\
=& \prod_{k=0}^{t-1} (1 + \mathbf{(1-\Psi_{j,t-k})} \gamma \eta_j s_{j,t-k}) A_{j0} \quad j = \{c, d\}
\end{aligned}
$$

Let $X_t = \lambda_t \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) L_{j,t} \hat{p}_{j,t}^{\frac{1}{1-\alpha}}$. Then the previous expression can be written as:

$$
\begin{aligned}
\mu_{j,t} =& X_t + \frac{A_{j,t+1}}{A_{j,t}} X_{t+1} + \frac{A_{j,t+1}}{A_{j,t}} \frac{A_{j,t+2}}{A_{j,t+1}} X_{t+2} + \frac{A_{j,t+1}}{A_{j,t}} \frac{A_{j,t+2}}{A_{j,t+1}} \frac{A_{j,t+3}}{A_{j,t+2}} X_{t+3} + ... \\
=& \left(\frac{1}{A_{j,t}}\right) (A_{j,t} X_t + A_{j,t+1} X_{t+1} + A_{j,t+2} X_{t+2} + ...)
\end{aligned}
$$

Plugging in for A's and X's this can be written as:

$$\mu_{j,t} = \left(\frac{1}{1 + \mathbf{(1-\Psi_{j,t})}\gamma\eta_j s_{j,t}}\right)\left(\frac{1}{A_{j,t-1}}\right)$$
$$\left(\lambda_t \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_{j,t}A_{j,t}\hat{p}_{j,t}^{\frac{1}{1-\alpha}} + \right.$$
$$\lambda_{t+1} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_{j,t+1}A_{j,t+1}\hat{p}_{j,t+1}^{\frac{1}{1-\alpha}} +$$
$$\left.\lambda_{t+2} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_{j,t+2}A_{j,t+2}\hat{p}_{j,t+2}^{\frac{1}{1-\alpha}} + \dots\right)$$
$$= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1-\alpha}{1 + \mathbf{(1-\Psi_{j,t})}\gamma\eta_j s_{j,t}}\right)\left(\frac{1}{A_{j,t-1}}\right)$$
$$\sum_{v\geq t}\lambda_v L_{jv}A_{jv}\hat{p}_{jv}^{\frac{1}{1-\alpha}}$$

So when does the social planner want to allocate scientists to the clean sector? Whenever the expected social gain from innovation in the clean sector, $\gamma\mathbf{(1-\Psi_{c,t})}\eta_c\mu_{c,t}A_{c,t-1}$, is larger than in the dirty sector:

$$\frac{\gamma\mathbf{(1-\Psi_{c,t})}\eta_c\mu_{c,t}A_{c,t-1}}{\gamma\eta_d\mu_{d,t}A_{d,t-1}} = \left(\frac{\eta_c}{\eta_d}\right)\mathbf{(1-\Psi_{c,t})}\left(\frac{1+\gamma\eta_d s_{d,t}}{1 + \mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}\right)\left(\frac{\sum_{v\geq t}\lambda_v L_{c,v}A_{c,v}\hat{p}_{c,v}^{\frac{1}{1-\alpha}}}{\sum_{v\geq t}\lambda_v L_{d,v}A_{d,v}\hat{p}_{d,v}^{\frac{1}{1-\alpha}}}\right) > 1 \tag{64}$$

which is an adjusted version of (A.14) in AABH. Furthermore, from (53) I know that $\lambda_t = \frac{1}{(1+\rho)^t}\frac{\partial u(C_t,S_t)}{\partial C_t}$ so that the latter becomes:

$$\frac{\eta_c\mu_{c,t}A_{c,t-1}}{\eta_d\mu_{d,t}A_{d,t-1}} = \left(\frac{\eta_c}{\eta_d}\right)\mathbf{(1-\Psi_{c,t})}\left(\frac{1+\gamma\eta_d s_{d,t}}{1 + \mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}\right)\left(\frac{\sum_{v\geq t}\frac{1}{(1+\rho)^v}\frac{\partial u(C_v,S_v)}{\partial C_v}L_{cv}A_{cv}\hat{p}_{cv}^{\frac{1}{1-\alpha}}}{\sum_{v\geq t}\frac{1}{(1+\rho)^v}\frac{\partial u(C_v,S_v)}{\partial C_v}L_{dv}A_{dv}\hat{p}_{dv}^{\frac{1}{1-\alpha}}}\right)$$

which is an adjusted version of (24) in AABH. The expression is not very intuitive to me so let us go back to (64) and rewrite it in a way allowing us to clearly see the difference between the privately and socially optimal allocation.

$$
\begin{aligned}
\frac{\eta_c \mu_{c,t} A_{c,t-1}}{\eta_d \mu_{d,t} A_{d,t-1}} &= \left(\frac{\eta_c}{\eta_d}\right) \mathbf{(1-\Psi_{c,t})} \left(\frac{1+\gamma\eta_d s_{d,t}}{1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}\right) \left(\frac{\lambda_t L_{c,t} A_{c,t} \hat{p}_{c,t}^{\frac{1}{1-\alpha}}}{\lambda_t L_{d,t} A_{d,t} \hat{p}_{d,t}^{\frac{1}{1-\alpha}}}\right) \left(\frac{\sum_{v>t} \lambda_v L_{cv} A_{cv} \hat{p}_{cv}^{\frac{1}{1-\alpha}}}{\sum_{v>t} \lambda_v L_{dv} A_{dv} \hat{p}_{dv}^{\frac{1}{1-\alpha}}}\right) \\
&= \left(\frac{\eta_c}{\eta_d}\right) \mathbf{(1-\Psi_{c,t})} \underbrace{\left(\frac{L_{c,t} \hat{p}_{c,t}^{\frac{1}{1-\alpha}} A_{c,t-1}}{L_{d,t} \hat{p}_{d,t}^{\frac{1}{1-\alpha}} A_{d,t-1}}\right)}_{\left(\frac{(1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}) A_{c,t-1}}{\left(1+\gamma\eta_d s_{d,t}\right) A_{d,t-1}}\right)^{-(1+\phi)} \left(\frac{A_{c,t-1}}{A_{d,t-1}}\right) \text{ from B.4.5}} \left(\frac{\sum_{v>t} \lambda_v L_{cv} A_{cv} \hat{p}_{cv}^{\frac{1}{1-\alpha}}}{\sum_{v>t} \lambda_v L_{dv} A_{dv} \hat{p}_{dv}^{\frac{1}{1-\alpha}}}\right) \\
&= \underbrace{\left(\frac{\eta_c}{\eta_d}\right) \mathbf{(1-\Psi_{c,t})} \left(\frac{1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}{1+\gamma\eta_d s_{d,t}}\right)^{-(1+\phi)} \left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)^{-\phi}}_{\text{Private value of innovation in clean vs. dirty}} \left(\frac{\sum_{v>t} \lambda_v L_{cv} A_{cv} \hat{p}_{cv}^{\frac{1}{1-\alpha}}}{\sum_{v>t} \lambda_v L_{dv} A_{dv} \hat{p}_{dv}^{\frac{1}{1-\alpha}}}\right)
\end{aligned}
$$

where I took the expression for $v = t$ out of the sum in the first step, substituted for $A_{c,t}$ and $A_{d,t}$ in the second step and substituted for an expression previously derived in Appendix B.4.5. This final expression corresponds to (18) in the text.

How can the government implement such an optimal allocation of scientists? First note that the shadow prices of dirty and clean inputs in the optimal allocation satisfy:

$$
\frac{\hat{p}_{c,t}}{\hat{p}_{d,t}} = \left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha} \tag{65}
$$

This comes from the FOC of the intermediate goods producers with respect to labour and the equilibrium demand for machines:

$$
\begin{aligned}
w_t =& (1-\alpha)\hat{p}_{j,t} L_{j,t}^{-\alpha} \int_0^1 A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha} di \\
=& (1-\alpha)\hat{p}_{j,t} L_{j,t}^{-\alpha} \int_0^1 A_{j,i,t} di \left(\frac{\alpha}{\psi}\hat{p}_{j,t}\right)^{\frac{\alpha}{1-\alpha}} L_{j,t}^{\alpha} \\
=& (1-\alpha) \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \hat{p}_{j,t}^{\frac{1}{1-\alpha}} A_{j,t} \\
\hat{p}_{j,t}^{\frac{1}{1-\alpha}} =& w_t (1-\alpha)^{-1} \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} A_{j,t}^{-1} \\
\frac{\hat{p}_{c,t}}{\hat{p}_{d,t}} =& \left(\frac{A_{d,t}}{A_{c,t}}\right)^{1-\alpha}
\end{aligned}
$$

Dividing (55) by (56) and using (57) I get :

$$\left(\frac{Y_{c,t}}{Y_{d,t}}\right)^{-\frac{1}{\varepsilon}} = \left(\frac{\hat{p}_{c,t}}{\hat{p}_{d,t}}\right)\left(\frac{1}{1+\tau_t}\right)$$

Plugging in (62) yields:

$$\left(\frac{L_{c,t}}{L_{d,t}}\right)^{-\frac{1}{\varepsilon}}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\frac{1}{\varepsilon}}\left(\frac{\hat{p_{c,t}}}{\hat{p_{d,t}}}\right)^{-\frac{\alpha}{(1-\alpha)\varepsilon}-1} = \left(\frac{1}{1+\tau_t}\right)$$

$$\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)\left(\frac{\hat{p_{c,t}}}{\hat{p_{d,t}}}\right)^{\frac{\alpha}{(1-\alpha)}+\varepsilon} = (1+\tau_t)^{\varepsilon}$$

Plugging in (65) yields:

$$\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)\left(\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-(1-\alpha)}\right)^{\frac{\alpha}{(1-\alpha)}+\varepsilon} = (1+\tau_t)^{\varepsilon}$$

$$\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\alpha-(1-\alpha)\varepsilon+1} = (1+\tau_t)^{\varepsilon}$$

$$\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t}}{A_{d,t}}\right)^{\phi} = (1+\tau_t)^{\varepsilon}$$

$$\frac{L_{c,t}}{L_{d,t}} = (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi} \tag{66}$$

where

$$\phi = (1-\varepsilon)(1-\alpha)$$

Next, using (58), I find that the monopolist's profits are:

$$\begin{aligned}
\pi_{j,i,t} &= (p_{j,i,t}-\psi)\left(\frac{\alpha}{\psi}\hat{p}_{j,t}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \\
&= \left(\frac{\psi}{\alpha}-\psi\right)\left(\frac{\alpha}{\psi}\hat{p}_{j,t}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \\
&= \psi\frac{(1-\alpha)}{\alpha}\left(\frac{\alpha}{\psi}\hat{p}_{j,t}\right)^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t} \\
&= (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\hat{p}_{j,t}^{\frac{1}{1-\alpha}} L_{j,t}A_{j,i,t}
\end{aligned}$$

Integrating over all machines, I get:

$$\int_0^1 \pi_{j,i,t} di = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \hat{p}_{j,t}^{\frac{1}{1-\alpha}} L_{j,t} \int_0^1 A_{j,i,t} di$$
$$\pi_{j,t} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \hat{p}_{j,t}^{\frac{1}{1-\alpha}} L_{j,t} A_{j,t}$$

Now remember that, conditional on success (i.e. with probability $(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{j,t}})\eta_j$), the scientists in sector j will run machines with an improved technology of $A_{j,t} = (1+\gamma)A_{j,t-1}$. Thus, expected period-profits in the clean sector for a *given* subsidy ν_t are:

$$\Pi_{c,t} = (1+\nu_t)(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\eta_c(1-\alpha)(1+\gamma)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \hat{p}_{c,t}^{\frac{1}{1-\alpha}} L_{c,t} A_{c,t-1} \tag{67}$$

C.3.1 Patent horizon = 1

Relative expected profits then become:

$$\begin{aligned}
\frac{\Pi_{c,t}}{\Pi_{d,t}} &= (1+\nu_t)\left(\frac{\eta_c}{\eta_d}\right)(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\left(\frac{\hat{p}_{c,t}}{\hat{p}_{d,t}}\right)^{\frac{1}{1-\alpha}}\left(\frac{L_{c,t}}{L_{d,t}}\right)\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right) \\
&= (1+\nu_t)\left(\frac{\eta_c}{\eta_d}\right)(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\left(\frac{A_{d,t}}{A_{c,t}}\right)(1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right) \\
&= (1+\nu_t)(1+\tau_t)^{\varepsilon}\left(\frac{\eta_c}{\eta_d}\right)(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-(1+\phi)}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)
\end{aligned} \tag{68}$$

With $A_{j,t} = (1+(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{j,t}})\gamma\eta_j s_{j,t})\, A_{j,t-1}$, I get:

$$\begin{aligned}
\frac{\Pi_{c,t}}{\Pi_{d,t}} &= (1+\nu_t)(1+\tau_t)^{\varepsilon}\left(\frac{\eta_c}{\eta_d}\right)(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\left(\frac{1+(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\gamma\eta_c s_{c,t}A_{c,t-1}}{1+\gamma\eta_d s_{d,t}A_{d,t-1}}\right)^{-(1+\phi)}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right) \\
&= (1+\nu_t)(1+\tau_t)^{\varepsilon}\left(\frac{\eta_c}{\eta_d}\right)(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\left(\frac{1+(\mathbf{1}-\mathbf{\Psi}_{\boldsymbol{c,t}})\gamma\eta_c s_{c,t}}{1+\gamma\eta_d s_{d,t}}\right)^{-(1+\phi)}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)^{-\phi}
\end{aligned} \tag{69}$$

The optimal subsidy ensures that whenever the optimal allocation involves $s_{c,t} > 0$, (69) evaluated at $s_{c,t} = s^*_{c,t}$ is greater than one. More formally:

$$
\begin{aligned}
\nu_t \geq \hat{\nu}_t \equiv & \left.\frac{\Pi_{d,t}}{\Pi_{c,t}}\right|_{s_{c,t}=s^*_{c,t}} - 1 \\
= & (1+\tau_t)^{\varepsilon}\left(\frac{\eta_d}{\eta_c}\right)\mathbf{(1-\Psi_{c,t})}^{-1}\left(\frac{1+\mathbf{(1-\Psi_{c,t})}\gamma\eta_c s_{c,t}}{1+\gamma\eta_d(1-s_{c,t})}\right)^{1+\phi}\left(\frac{A_{c,t-1}}{A_{d,t-1}}\right)^{\phi} - 1
\end{aligned}
$$

This corresponds to (19) in the text. The minimum clean research subsidy, $\hat{\nu}_t$, corresponds to the relative excess profits in the dirty sector if the scientist chose to go there. Since the subsidy will only be in place whenever the socially efficient allocation involves at least some research in the clean sector, i.e. $s_{c,t} > 0$, such a subsidy ensures that profits in the clean sector are always strictly larger than profits in the dirty sector when it is in place. Note that if I set $\kappa = 0$, we are back to the baseline case and the expression simplifies to (18) in AABH.

C.4 Equilibrium

Here, I solve for the socially optimal level of employment and prices in each sector. Then, I solve for aggregate output and consumption as well as the period-zero level of productivity to match the given initial levels of dirty and clean input production (calibrated with US data) in AABH's Matlab code.

C.4.1 Prices

First, I solve the adjusted problem of the final good producer. Remember that the final good is the numéraire and thus has a price equal to 1. The final good producer maximizes in the presence of a carbon tax:

$$
\left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} - p_{c,t}Y_{c,t} - (1+\tau_t)p_{d,t}Y_{d,t}
$$

The first-order conditions for the clean and dirty input respectively are:

$$
\begin{aligned}
Y_{c,t} &= p_{c,t}^{-\varepsilon} Y_t \\
Y_{d,t} &= ((1+\tau_t)p_{d,t})^{-\varepsilon} Y_t
\end{aligned}
$$

Perfect competition in the final good sector implies that firms make zero profits in the presence of the carbon tax. Therefore:

$$
0 \quad = \quad \underbrace{\left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}}_{Y_t} - p_{c,t}Y_{c,t} - (1+\tau_t)p_{d,t}Y_{d,t}
$$

Then, plugging in the previously obtained first-order conditions, I get:

$$\begin{aligned}
0 =& Y_t - p_{c,t}^{1-\varepsilon} Y_t - ((1+\tau_t)p_{d,t})^{1-\varepsilon} Y_t \\
1 =& p_{c,t}^{1-\varepsilon} + ((1+\tau_t)p_{d,t})^{1-\varepsilon} \\
1 =& \left[p_{c,t}^{1-\varepsilon} + (1+\tau_t)^{1-\varepsilon} p_{d,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \qquad (70)
\end{aligned}$$

I now combine (65) and (70):

$$\begin{aligned}
1 =& \left(\left(\frac{A_{c,t}}{A_{d,t}} \right)^{-(1-\alpha)} \hat{p}_{d,t} \right)^{1-\varepsilon} + (1+\tau_t)^{1-\varepsilon} \hat{p}_{d,t}^{1-\varepsilon} \\
=& \hat{p}_{d,t}^{1-\varepsilon} \left(\left(\frac{A_{d,t}}{A_{c,t}} \right)^{\phi} + (1+\tau_t)^{1-\varepsilon} \right) \\
=& \hat{p}_{d,t}^{1-\varepsilon} \left(\frac{(1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} + A_{d,t}^{\phi}}{A_{c,t}^{\phi}} \right) \\
\hat{p}_{d,t} =& \left(\frac{A_{c,t}^{\phi}}{(1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi} + A_{d,t}^{\phi}} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}$$

Given that:

$$\frac{1-\alpha}{(1-\varepsilon)(1-\alpha)} = \frac{1}{1-\varepsilon} = \frac{1-\alpha}{\phi}$$

I can rewrite the previous expression as:

$$\begin{aligned}
\hat{p}_{d,t} =& \left(\frac{A_{c,t}^{\phi}}{(1+\tau_t)^{\frac{\phi}{1-\alpha}} A_{c,t}^{\phi} + A_{d,t}^{\phi}} \right)^{\frac{1-\alpha}{\phi}} \\
=& A_{c,t}^{1-\alpha} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A_{c,t}^{\phi} + A_{d,t}^{\phi} \right)^{\frac{\alpha-1}{\phi}} \\
\hat{p}_{d,t}^{\frac{1}{1-\alpha}} =& A_{c,t} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A_{c,t}^{\phi} + A_{d,t}^{\phi} \right)^{-\frac{1}{\phi}} \qquad (71)
\end{aligned}$$

Now, from (65) I know that:

$$
\begin{aligned}
\hat{p}_{c,t}^{\frac{1}{1-\alpha}} &= \left(\frac{A_{d,t}}{A_{c,t}}\right)\hat{p}_{d,t}^{\frac{1}{1-\alpha}} \\
&= \left(\frac{A_{d,t}}{A_{c,t}}\right)A_{c,t}\left((1+\tau_t)^{\frac{\phi}{1-\alpha}}A_{c,t}^{\phi}+A_{d,t}^{\phi}\right)^{-\frac{1}{\phi}} \\
&= A_{d,t}\left((1+\tau_t)^{\frac{\phi}{1-\alpha}}A_{c,t}^{\phi}+A_{d,t}^{\phi}\right)^{-\frac{1}{\phi}} \qquad (72)
\end{aligned}
$$

C.4.2 Labour

I first combine (66) with (5):

$$
\begin{aligned}
\left(\frac{L_{c,t}}{L_{d,t}}\right) &= (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi} \\
L_{c,t}+L_{d,t} &= 1
\end{aligned}
$$

to obtain:

$$
\begin{aligned}
L_{c,t} &= (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}(1-L_{c,t}) \\
L_{c,t}\left(1+(1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}\right) &= (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi} \\
L_{c,t}\left(\frac{A_{c,t}^{\phi}+(1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}}{A_{c,t}^{\phi}}\right) &= (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi} \\
L_{c,t} &= (1+\tau_t)^{\varepsilon}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi} \\
&\quad \left(\frac{A_{c,t}^{\phi}}{A_{c,t}^{\phi}+(1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}}\right) \\
&= (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi} \\
&\quad \left(A_{c,t}^{\phi}+(1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}\right)^{-1} \qquad (73)
\end{aligned}
$$

And similarly:

$$
\begin{aligned}
L_{d,t} =& 1 - L_{c,t} \\
=& 1 - (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
=& \frac{\left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right) - (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}}{\left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)} \\
=& A_{c,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1}
\end{aligned}
\tag{74}
$$

C.4.3 Output

I can now derive equilibrium production levels using (62) and plugging in from (71), (72), (73) and (74):

$$
\begin{aligned}
Y_{c,t} =& L_{c,t} A_{c,t} \left(\frac{\alpha \hat{p}_{c,t}}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} \\
& A_{c,t} \left(\frac{\alpha \left(A_{d,t} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A_{c,t}^{\phi} + A_{d,t}^{\phi} \right)^{-\frac{1}{\phi}} \right)^{1-\alpha}}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& (1+\tau_t)^{\varepsilon} \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t}^{\alpha+\phi} [A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}]^{-\frac{\alpha}{\phi}} \\
& \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1}
\end{aligned}
$$

$$
\begin{aligned}
Y_{d,t} =& L_{d,t} A_{d,t} \left(\frac{\alpha \hat{p}_{d,t}}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& A_{c,t}^{\phi} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1} A_{d,t} \\
& \left(\frac{\alpha \left(A_{c,t} [(1+\tau_t)^{\frac{\phi}{1-\alpha}} A_{c,t}^{\phi} + A_{d,t}^{\phi}]^{-\frac{1}{\phi}} \right)^{1-\alpha}}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} A_{d,t} A_{c,t}^{\alpha+\phi} [A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}]^{-\frac{\alpha}{\phi}} \\
& \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi} \right)^{-1}
\end{aligned}
$$

To find equilibrium aggregate production, express clean output as a function of dirty output by plugging $L_{c,t} = (1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-\phi} L_{d,t}$ and $\hat{p}_{c,t} = \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-(1-\alpha)} \hat{p}_{d,t}$ into (62).

$$
\begin{aligned}
Y_{c,t} =& (1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-\phi} L_{d,t} A_{c,t} \left(\frac{\alpha \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-(1-\alpha)} \hat{p}_{d,t}}{\psi} \right)^{\frac{\alpha}{1-\alpha}} \\
=& (1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{1-\phi-\alpha} \underbrace{L_{d,t} A_{d,t} \left(\frac{\alpha \hat{p}_{d,t}}{\psi} \right)^{\frac{\alpha}{1-\alpha}}}_{Y_{d,t}} \qquad (75)
\end{aligned}
$$

Now plug (75) into $Y_t = \left(Y_{c,t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$

$$
\begin{aligned}
Y_t =& \left(\left((1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-(\phi+\alpha-1)} Y_{d,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} + Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
=& \left(Y_{d,t}^{\frac{\varepsilon-1}{\varepsilon}} \left(1 + \left((1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-(\phi+\alpha-1)} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right) \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
=& Y_{d,t} \left(1 + \left((1+\tau_t)^{\varepsilon} \left(\frac{A_{c,t}}{A_{d,t}} \right)^{-(\phi+\alpha-1)} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}
$$

Since $\frac{\varepsilon-1}{\varepsilon} = \frac{\phi}{\phi+\alpha-1}$, the former becomes:

$$
\begin{aligned}
Y_t =& Y_{d,t}\left(1+(1+\tau_t)^{\varepsilon-1}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}\right)^{\frac{\varepsilon}{\varepsilon-1}}\\
=& Y_{d,t}\left(\frac{A_{c,t}^{\phi}+A_{d,t}^{\phi}(1+\tau_t)^{\varepsilon-1}}{A_{c,t}^{\phi}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\\
=& Y_{d,t}\left(\frac{A_{c,t}^{\phi}+A_{d,t}^{\phi}(1+\tau_t)^{\varepsilon-1}}{A_{c,t}^{\phi}}\right)^{\frac{\phi+\alpha-1}{\phi}}\\
=& Y_{d,t}\left(A_{c,t}^{\phi}+A_{d,t}^{\phi}(1+\tau_t)^{\varepsilon-1}\right)^{\frac{\phi+\alpha-1}{\phi}}A_{c,t}^{1-\phi-\alpha}
\end{aligned}
$$

Now plug in for $Y_{d,t}$:

$$\begin{aligned} Y_t &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d,t} A_{c,t}^{\alpha+\phi} [A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}]^{-\frac{\alpha}{\phi}} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}\right)^{-1} \left(A_{c,t}^{\phi} + A_{d,t}^{\phi} (1+\tau_t)^{\varepsilon-1}\right)^{\frac{\phi+\alpha-1}{\phi}} A_{c,t}^{1-\phi-\alpha} \\ &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d,t} A_{c,t} [A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon} A_{c,t}^{\phi}]^{-\frac{\alpha}{\phi}} \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon} A_{d,t}^{\phi}\right)^{-1} \left(A_{c,t}^{\phi} + A_{d,t}^{\phi} (1+\tau_t)^{\varepsilon-1}\right)^{\frac{\phi+\alpha-1}{\phi}} \end{aligned} \tag{76}$$

C.4.4 Consumption

To find C_t, use the resource constraint (6):

$$C_t + \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) = Y_t$$

as well as (58):

$$x_{j,i,t} = \left(\frac{\alpha}{\psi} \hat{p}_{j,t}\right)^{\frac{1}{1-\alpha}} L_{j,t} A_{j,i,t}$$

$$\begin{aligned} C_t &= Y_t - \psi \left(\int_0^1 \left(\frac{\alpha}{\psi}\hat{p}_{c,t}\right)^{\frac{1}{1-\alpha}} L_{c,t} A_{cit} di + \int_0^1 \left(\frac{\alpha}{\psi}\hat{p}_{d,t}\right)^{\frac{1}{1-\alpha}} L_{d,t} A_{dit} di \right) \\ &= Y_t - \psi \left(\left(\frac{\alpha}{\psi}\hat{p}_{c,t}\right)^{\frac{1}{1-\alpha}} L_{c,t} \int_0^1 A_{cit} di + \left(\frac{\alpha}{\psi}\hat{p}_{d,t}\right)^{\frac{1}{1-\alpha}} L_{d,t} \int_0^1 A_{dit} di \right) \\ &= Y_t - \psi \left(\left(\frac{\alpha}{\psi}\hat{p}_{c,t}\right)^{\frac{1}{1-\alpha}} L_{c,t} A_{c,t} + \left(\frac{\alpha}{\psi}\hat{p}_{d,t}\right)^{\frac{1}{1-\alpha}} L_{d,t} A_{d,t} \right) \\ &= Y_t - \alpha \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \left(\hat{p}_{c,t}^{\frac{1}{1-\alpha}} L_{c,t} A_{c,t} + \hat{p}_{d,t}^{\frac{1}{1-\alpha}} L_{d,t} A_{d,t} \right) \end{aligned}$$

Now I plug in from (71), (72), (73) and (74):

$$\begin{aligned}
C_t =& Y_t - \alpha \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \Bigg(A_{d,t} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)^{-\frac{1}{\phi}} (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t} \left(A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t}\right)^{-1} \\
& A_{c,t} + A_{c,t} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)^{-\frac{1}{\phi}} A^{\phi}_{c,t} \left(A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t}\right)^{-1} A_{d,t} \Bigg) \\
=& Y_t - \alpha \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)^{-\frac{1}{\phi}} \left(A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t}\right)^{-1} \left((1+\tau_t)^{\varepsilon} A^{\phi}_{d,t} + A^{\phi}_{c,t}\right) \\
=& Y_t - \alpha \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t} \left((1+\tau_t)^{\frac{\phi}{1-\alpha}} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)^{-\frac{1}{\phi}}
\end{aligned}$$

Now I plug in from (76):

$$\begin{aligned}
C_t =& \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d,t} A_{c,t} [A^{\phi}_{d,t} + (1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t}]^{-\frac{\alpha}{\phi}} \left(A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t}\right)^{-1} \left(A^{\phi}_{c,t} + A^{\phi}_{d,t} (1+\tau_t)^{\varepsilon-1}\right)^{\frac{\phi+\alpha-1}{\phi}} \\
& - \alpha \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t} \left((1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)^{-\frac{1}{\phi}} \\
=& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t}}{\left(A^{\phi}_{d,t} + (1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t}\right)^{\frac{1}{\phi}}} \left(\frac{\left(A^{\phi}_{d,t} + (1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t}\right)^{\frac{1-\alpha}{\phi}} \left(A^{\phi}_{c,t} + A^{\phi}_{d,t} (1+\tau_t)^{\varepsilon-1}\right)^{\frac{\phi+\alpha-1}{\phi}}}{A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t}} - \alpha \right)
\end{aligned}$$

Note that: $\frac{1-\alpha}{\phi} = \frac{1-\alpha}{(1-\alpha)(1-\varepsilon)} = \frac{1}{1-\varepsilon}$ and $\frac{\phi+\alpha-1}{\phi} = 1 + \frac{-(1-\alpha)}{(1-\alpha)(1-\varepsilon)} = 1 - \frac{1}{1-\varepsilon}$.

$$C_t = \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t} A_{d,t}}{\left(A^{\phi}_{d,t} + (1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t}\right)^{\frac{1}{\phi}}} \left(\frac{\left(A^{\phi}_{d,t} + (1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t}\right)^{\frac{1}{1-\varepsilon}} \left(A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon-1} A^{\phi}_{d,t}\right)^{1-\frac{1}{1-\varepsilon}}}{A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon} A^{\phi}_{d,t}} - \alpha \right)$$

Note that: $\left(A^{\phi}_{c,t} + (1+\tau_t)^{\varepsilon-1} A^{\phi}_{d,t}\right)^{1-\frac{1}{1-\varepsilon}} = \left(\left((1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)(1+\tau_t)^{\varepsilon-1}\right)^{1-\frac{1}{1-\varepsilon}} = \left((1+\tau_t)^{1-\varepsilon} A^{\phi}_{c,t} + A^{\phi}_{d,t}\right)^{1-\frac{1}{1-\varepsilon}} (1+\tau_t)^{\varepsilon}$

$$
\begin{aligned}
C_t =& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t}A_{d,t}}{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{\phi}}} \left(\frac{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{1-\varepsilon}} \left((1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi} + A_{d,t}^{\phi}\right)^{1-\frac{1}{1-\varepsilon}} (1+\tau_t)^{\varepsilon}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}} - \alpha \right) \\
=& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t}A_{d,t}}{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{\phi}}} \left(\frac{\left((1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi} + A_{d,t}^{\phi}\right)(1+\tau_t)^{\varepsilon}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}} - \alpha \right)
\end{aligned}
$$

Note that: $(1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi} + A_{d,t}^{\phi} = \left((1+\tau_t)A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}\right)(1+\tau_t)^{-\varepsilon}$

$$
\begin{aligned}
C_t =& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t}A_{d,t}}{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{\phi}}} \left(\frac{\left((1+\tau_t)A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}\right)(1+\tau_t)^{-\varepsilon}(1+\tau_t)^{\varepsilon}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}} - \alpha \right) \\
=& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t}A_{d,t}}{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{\phi}}} \left(\frac{\tau_t A_{c,t}^{\phi} + \left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}\right)}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}} - \alpha \right) \\
=& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t}A_{d,t}}{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{\phi}}} \left(\frac{\left(A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}\right)\left(1 + \frac{\tau_t A_{c,t}^{\phi}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}}\right)}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}} - \alpha \right) \\
=& \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c,t}A_{d,t}}{\left(A_{d,t}^{\phi} + (1+\tau_t)^{1-\varepsilon}A_{c,t}^{\phi}\right)^{\frac{1}{\phi}}} \left(1 - \alpha + \frac{\tau_t A_{c,t}^{\phi}}{A_{c,t}^{\phi} + (1+\tau_t)^{\varepsilon}A_{d,t}^{\phi}} \right) \qquad (77)
\end{aligned}
$$

which corresponds to the expression in AABH's Matlab code.

C.4.5 Initial productivity

I need the initial levels of productivity to match the given initial levels of dirty and clean input production. So I ultimately need to express both A_{c0} and A_{d0} in terms of Y_{c0} and Y_{d0}. Note that in period zero, there is no carbon tax, i.e. $\tau_0 = 0$.

First, I divide the equilibrium expressions for clean and dirty output from the social planner problem:

$$
\begin{aligned}
\frac{Y_{c0}}{Y_{d0}} &= \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{c0} A_{d0}^{\alpha+\phi} [A_{d0}^{\phi} + A_{c0}^{\phi}]^{-\frac{\alpha}{\phi}} \left(A_{c0}^{\phi} + A_{d0}^{\phi}\right)^{-1}}{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d0} A_{c0}^{\alpha+\phi} [A_{d0}^{\phi} + A_{c0}^{\phi}]^{-\frac{\alpha}{\phi}} \left(A_{c0}^{\phi} + A_{d0}^{\phi}\right)^{-1}} \\
&= \left(\frac{A_{c0}}{A_{d0}}\right)^{1-\alpha-\phi} \\
A_{c0} &= \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0} \qquad (78)
\end{aligned}
$$

Now I plug the latter into the equilibrium level of dirty input production from the social planner problem:

$$
\begin{aligned}
Y_{d0} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d0} \left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0}\right)^{\alpha+\phi} \left(A_{d0}^{\phi} + \left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0}\right)^{\phi}\right)^{-\frac{\alpha}{\phi}} \left(\left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0}\right)^{\phi} + A_{d0}^{\phi}\right)^{-1} \\
&= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d0} \left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0}\right)^{\alpha+\phi} \left(A_{d0}^{\phi} + \left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0}\right)^{\phi}\right)^{-\frac{\alpha+\phi}{\phi}} \\
&= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d0} \left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1}{1-\alpha-\phi}} A_{d0}\right)^{\alpha+\phi} \left(A_{d0}^{\phi} \left(1 + \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{\phi}{1-\alpha-\phi}}\right)\right)^{-\frac{\alpha+\phi}{\phi}} \\
&= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{d0}^{1+\alpha+\phi-\alpha-\phi} \left(\left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{-\phi}{1-\alpha-\phi}}\right)^{-\frac{\alpha+\phi}{\phi}} \left(1 + \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{\phi}{1-\alpha-\phi}}\right)^{-\frac{\alpha+\phi}{\phi}} \\
A_{d0} &= \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{d0} \left(\left(1 + \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{\phi}{1-\alpha-\phi}}\right) \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{-\phi}{1-\alpha-\phi}}\right)^{\frac{\alpha}{\phi}+1} \\
&= \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{d0} \left(1 + \left(\frac{Y_{d0}}{Y_{c0}}\right)^{\frac{\phi}{1-\alpha-\phi}}\right)^{\frac{\alpha}{\phi}+1}
\end{aligned}
$$

Note that: $\frac{\phi}{1-\alpha-\phi} = \frac{(1-\alpha)(1-\varepsilon)}{1-\alpha-(1-\alpha)(1-\varepsilon)} = \frac{(1-\alpha)(1-\varepsilon)}{(1-\alpha)(1-(1-\varepsilon))} = \frac{1-\varepsilon}{\varepsilon}$

$$A_{d0} = \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{d0} \left(1 + \left(\frac{Y_{d0}}{Y_{c0}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right)^{\frac{\alpha}{\phi}+1} \tag{79}$$

which corresponds to the expression in the AABH's Matlab code. Now, by symmetry of the expression for relative output, I can infer that the corresponding expression for initial clean productivity is:

$$A_{c0} = \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{c0} \left(1 + \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right)^{\frac{\alpha}{\phi}+1} \tag{80}$$

which corresponds to the expression in AABH's Matlab code.